Heat and Mass Transfer in MHD Casson Fluid Flow along Exponentially Permeable Stretching Sheet in Presence of Radiation and Chemical Reaction

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Abstract The present paper is concerned with the study of heat and mass transfer in MHD Casson fluid flow along exponentially permeable stretching sheet in presence of radiation and chemical reaction. The resulting momentum, energy and concentration equations are then made similar by introducing the usual similarity transformations is used to convert the governing partial differential equations into a system of coupled non-linear differential equations. The resulting coupled non-linear differential equations are solved numerically by using MATLAB bvp4c package. The effects of various non-dimensional governing parameters on velocity, temperature and concentration profiles are discussed and presented graphically.

Keywords: MHD, Heat and Mass Transfer, Casson fluid flow, Radiation, , Chemical reaction, Heat source.

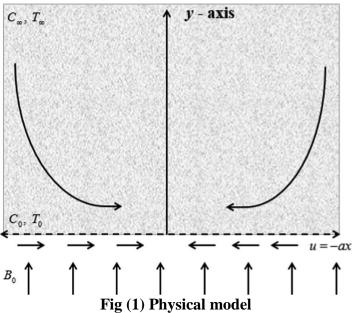
Introduction

During the past years, MHD flow, heat and mass transfer problems have become more important in many engineering and industrial applications. These include Magnetohydrodynamic power generators and accelerators, cooling of nuclear reactors, and crystal growth. Such problem has gained great attention among researchers because of its wide applications in various areas. Mangathaia et al. [1] examined MHD free flow past a vertical porous plate in presence of radiation and heat generation. Raju et al. [2] elaborated heat and mass transfer in MHD Casson fluid flow over an exponentially permeable stretching surface. Muhammad et al. [3] analyzed heat and mass transfer for the MHD of Casson fluid through porous medium over shrinking. Malik et al. [4] has studied MHD 3D Maxwell fluid flow towards a horizontal stretched surface with convective wall. Nadeem et al. [5] observed MHD flow of a Casson fluid over an exponentially shrinking sheet. Malik et al. [6] examined the boundary layer flow of Casson nanofluid over a vertical exponentially stretching cylinder. Animasaun et al. [7] elaborated Casson fluid flow with variable thermo-physical property along exponentially stretching sheet with suction and exponentially decaying internal heat generation using the homotopy analysis method. Chen et al. [8] observed heat transfer of a continuous stretching surface with suction or blowing. Shankar et al. [9] examined the joule heating effect on MHD natural convective fluid flow in a permeable medium over a semi-infinite inclined vertical plate in the presence of the chemical reaction. Shankar et al. [10] has studied radiation effect on MHD boundary layer flow due to an exponentially stretching sheet. Several attempts have been made to analyze the effect of thermal radiation and chemical reaction under various physical situations (see [11-33]).

The aim of the current model is to analyze the study of Heat and mass transfer in MHD Casson fluid flow along exponentially permeable stretching sheet in presence of radiation and chemical reaction. Appropriate similarity transformation is used to convert the governing partial differential equations into a system of coupled non-linear differential equations. The resulting coupled non-linear differential equations are solved numerically by using MATLAB bvp4c package. The effects of various pertinent parameters on the momentum, heat and mass transfer characteristics have been studied and numerical result are presented graphically.

2. Formulation of the problem:

Let us assume the rheology of hydro-magnetic non- Newtonian fluid over a permeable wall which undergoes shrinking phenomenon. The wall permeability properties are engendered in order to analyze the blowing and suction processes. The non-Newtonian fluid drenches the porous medium y > 0 whereas the flow dynamics occurs in the region y > 0 as depicted in the Fig. 1.



Presence of the rheological equation of state for an isotropic and incompressible flow of a Casson flow of a Casson fluid is as follows:

$$\tau_{ab} = \begin{cases} 2\left(\mu_{\rm B} + P_{\rm y} / (2\pi)^{1/2}\right)e_{ab}, & \pi > \pi_{c} \\ 2\left(\mu_{\rm B} + P_{\rm y} / (2\pi)^{1/2}\right)e_{ab}, & \pi < \pi_{c} \end{cases}$$

where $\pi = e_{ab}e_{ab}$ is the square of deformation rate, π_c is the critical value, μ_B stands for dynamic plastic viscosity.

The governing equations the flow can be written as:

$$\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} = 0,\tag{1}$$

$$\beta \left(v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} \right) - v \left(1 + \beta \right) \frac{\partial^2 u}{\partial y^2} + \frac{v \beta}{k} u + \frac{\sigma B_0^2 \beta}{\rho} U = 0, \tag{2}$$

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$$\rho c_{p} u \frac{\partial T}{\partial x} + \rho c_{p} v \frac{\partial T}{\partial y} = k^{*} \frac{\partial^{2} T}{\partial y^{2}} - Q \left(T - T_{\infty} \right) - \frac{\partial q_{r}}{\partial y} + \mu \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial u}{\partial y} \right)^{2}, \tag{3}$$

$$v\frac{\partial C}{\partial y} + u\frac{\partial C}{\partial X} = -k_1 C + D\frac{\partial^2 C}{\partial y^2},\tag{4}$$

along with the permeable wall properties

$$u = -cx, v = V_0, T = T_0, C = C_0 \text{ at } y = 0,$$

$$u \to 0, C = C_0, T = T_0 \text{ as } v \to \infty,$$
(5)

where *u* and *v* stand for the velocities, *v* represents kinematic viscosity, β is parameter of the non-Newtonian fluid σ is the electrical conductivity, B_0 represents the magnetic field strength, ρ is the fluid density, *k* represents porous medium permeability, *T* stands for temperature, *Q* represents internal heat generation/absorption quantity, and $\lambda_1 = Q/c\rho c_\rho$ is the dimensionless form of heat generation (>0) and absorption (<0).*C* is the phenomenon of concentration, k^* describes thermal conductivity, *D* describes diffusion, k_1 is the chemical reaction. The suction as well as blowing properties are considered such that phenomenon of suction occurs when $V_0 < 0$ while $V_0 > 0$ represents blowing situation.

The radiative heat flux qr under Rosseland approximation has the form

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*}T^3\frac{\partial T}{\partial y}$$
(6)

Then equation(3) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{16\sigma^* T^3}{3k^* (\rho C_p)}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_f}{(\rho C_p)} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{(\rho C_p)} \left(T - T_{\infty}\right)$$
(7)

The following variables are utilized for simplifications

$$u = cxf(\eta), \quad v = -(cv)^{1/2} f(\eta), \eta = \sqrt{\frac{c}{v}} y,$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_0 - T_{\infty}}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_0 - C_{\infty}}.$$
(8)

Using equations (5)(7) and (8), equations (2)-(4) transformed to the ordinary differential equations if of the form

$$\left(1+\frac{1}{\beta}\right)f''' - \left(f'\right)^2 + ff'' - M^2 f' - Kf' = 0,$$
(9)

$$\frac{1}{\Pr}\left(\frac{4}{3}R+1\right)\theta'' + N\left(4f'\theta - f\theta'\right) + \left(1 + \frac{1}{\beta}\right)Ecf''^2 - Q\theta = 0$$
⁽¹⁰⁾

$$\phi'' + Scf \phi' - Sc\gamma \phi = 0. \tag{11}$$

The conditions at the boundary are given by

$$f(0) = S, f'(0) = -1, \theta(0) = \phi(0) = 1,$$

$$f(\infty) = \theta(\infty) = \phi(\infty) = 0,$$

$$(12)$$

where β represents the non-Newtonian parameter, M stands for magnetic parameter, λ_1

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describes parameter of heat source/sink, λ stands for the parameter of chemical reaction, K_1 is the parameter of porosity, while *Sc* and Pr represent the Schmidt and Prandtl number respectively, and *S* represents suction/blowing parameter. Their mathematical forms are given by

$$\beta = \mu_B \sqrt{2\pi_c} / p_y, M^2 = \frac{\sigma B_0^2}{cp}, K = \frac{v}{kc}, \Pr = \frac{v}{\alpha_m},$$

$$\lambda_1 = \frac{Q}{C\rho C_p}, Sc = \frac{v}{D}, \gamma = \frac{k_1}{c}, S = \frac{-V_0}{\sqrt{vc}}$$
(13)

The parameters of physical curiosity are local Nusselt number Nu_x , skin friction C_f and local Sherwood number *Sh*. These are described as

$$C_{f} = \frac{\tau_{w}}{\rho u_{w}^{2}}, N u_{x} = \frac{x q_{w}}{k^{*} (T_{w} - T_{\infty})}, S h = \frac{x j_{w}}{D(C_{w} - C_{\infty})},$$
(14)

Where τ_w represents the shear stress along the shrinking wall, q_w represents heat flux and j_w is wall mass transfer. These are described as

$$q_{w} = -k^{*} \left(\frac{\partial T}{\partial y}\right)_{y=0}, \pi_{w} = \left(\mu_{B} + p_{y} / \sqrt{2\pi}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}, j_{w} = -D \left(\frac{\partial C}{\partial y}\right)_{y=0}$$
(15)

The physical quantities of interest are the local skin friction coefficient, the wall heat transfer coefficient and mass transfer coefficients are given by,

$$C_{f}\left(\operatorname{Re}_{x}^{1/2}\right) = \left(1 + \frac{1}{\beta}\right) f''(0),$$

$$Nu_{x} / \sqrt{\operatorname{Re}_{x}} = -\theta'(0),$$

$$Sh / \sqrt{\operatorname{Re}_{x}} = -\phi'(0).$$
(16)

Results and discussion:

The system of nonlinear ordinary differential equations (9) to (11) with the boundary conditions (12) are solved numerically by using bvp4c with MATLAB package. The obtained results show the effects of the various non-dimensional governing parameters, namely exponential parameter (*N*), Magnetic field parameter (*M*), radiation parameter (*R*), Eckert number (*Ec*), heat source parameter (*Q*), Casson fluid parameter (β), Porosity parameter(K), Prandtl number (Pr), Schmidt number (Sc) and chemical results we used M=0.5; β =0.5; K=0.5; Pr=0.71; R=0.5; N=1; Ec=0.1; Q=1; Sc=0.6; γ =0.5; S=0.5. These values are treated as common throughout the study except the varied values in respective figures.

Fig. 2, 3 and 4 display the velocity, temperature and concentration profiles for different values of M when the other parameters are fixed. An applied of M within boundary layer has produced resistive-type force, which known as Lorentz force. This force act to retard the fluid motion along surface and simultaneously increase its temperature and concentration values. Therefore, one can see that the velocity boundary layer thickness decreases with the increase of M as shown

in Fig. 2. However, the temperature and concentration increase with the increasing of M shown in Fig. 3 and Fig.4. Figure 5 show the impact of K on temperature profiles. Increases K, increases temperature profile. Porosity is defined as the measure of void (or empty) spaces in a porous medium and is a fraction of the volume of voids over the total volume. Convection flows are often influenced by porosity and in result raise the fluid temperature. The influences of Pr on temperature are represented in the Fig. 6 which elucidates that both numerical solutions showing similar behavior. The Pr decays the temperature profile due to dominating impact of thermal diffusivity in the flow problem. The influence of R on temperature profiles is shown in Fig. 7. It can be observed that there is an enhancement in the temperature profiles when there is an increase in the radiation parameter. This may happen due to the fact that an increase in R enhances the thermal boundary layer. Fig.8 depict the influence of N on temperature profile. It is evident from the figure that an increase in N declines temperature. This happens due to the decreasing nature of the thermal profile with the increasing value of N. This may happen due to a decrease in the wall temperature throughout the boundary layer for positive values of N. The influence of Ec on the temperature profile is shown in Fig. 9. It is observed from the figure that an increase in the Ec enhances the flow and thermal boundary layer thickness. This is due to the fact that an increase in dissipation improves the thermal conductivity of the flow. This helps to enhance the momentum boundary layers. Fig 10, displays the effect of Q on temperature profiles of the flow. It is noticed that an increase in Q reduces the temperature profiles of the flow. It is expected that an increase in Q will release the heat energy to the flow, which causes the temperature profiles to enhance. But due to the domination of the external heat compared with Q supplied to the flow, we noticed reverse results to that of expected results. The figure 11, display the concentration profile for different values of Sc. It can be observed that the concentration profile is decreases with the increases of Sc. Chemical reaction parameter effect on concentration profile is shown in Fig. 12. It can be observed from the figure that increase in the value of γ reduces concentration. Due to an increase in the interfacial mass transfer we observed fall in concentration profile.

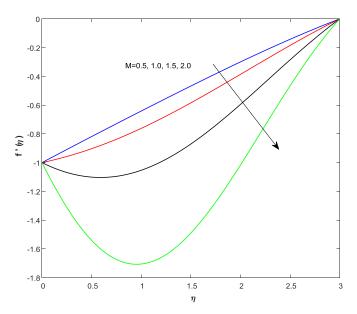


Fig.2. Velocity profiles for different values of *M*.

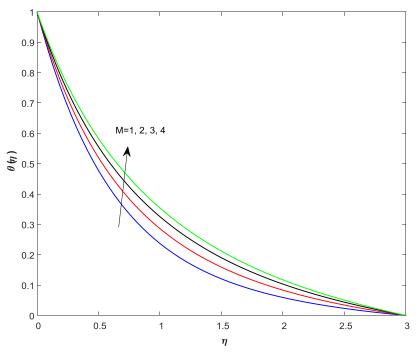


Fig.3. Temperature profiles for different values of *M*.

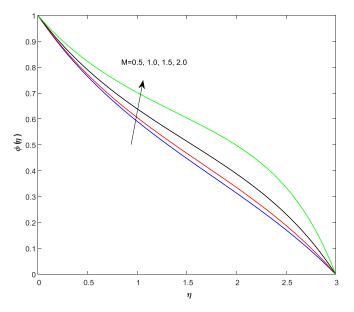


Fig.4. Concentration profiles for different values of *M*.

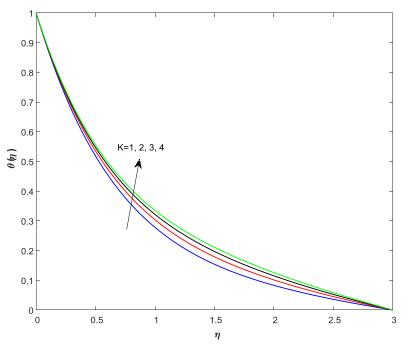


Fig.5. Temperature profiles for different values of K.

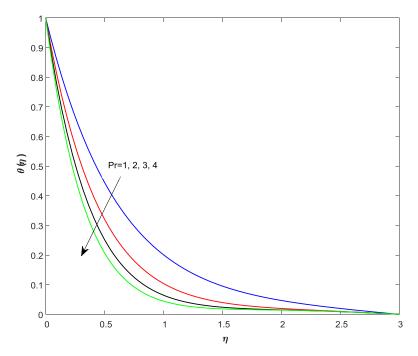


Fig.6. Temperature profiles for different values of Pr.

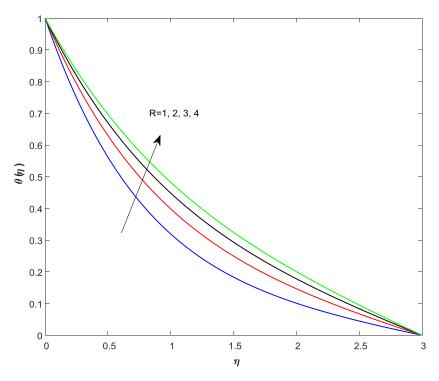


Fig.7. Temperature profiles for different values of R.

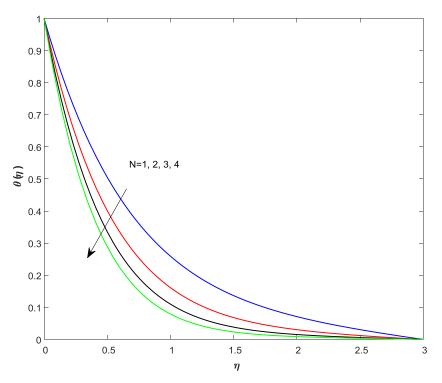


Fig.8. Temperature profiles for different values of N.

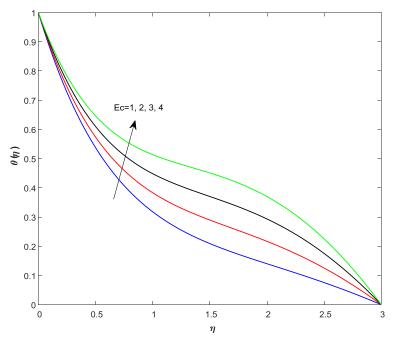


Fig.9. Temperature profiles for different values of Ec.

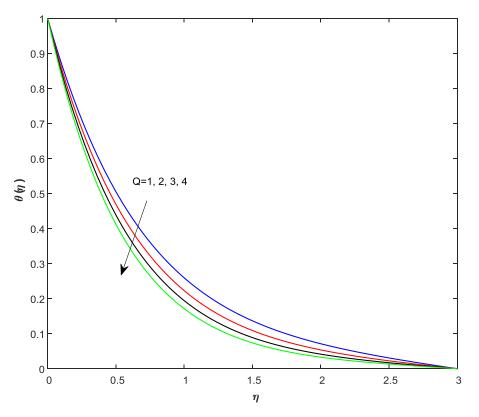


Fig.10. Temperature profiles for different values of Q.

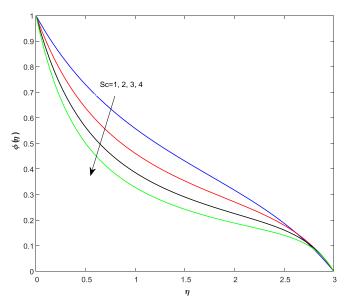


Fig.11. Concentration profiles for different values of Sc.

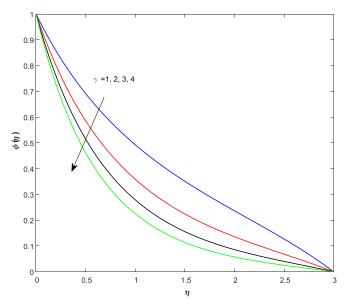


Fig.12. Concentration profiles for different values of γ .

Conclusions:

This paper presents the study of Heat and mass transfer in MHD Casson fluid flow along exponentially permeable stretching sheet in presence of radiation and chemical reaction. Appropriate similarity transformation is used to convert the governing partial differential equations into a system of coupled non-linear differential equations. The resulting coupled non-linear differential equations are solved numerically by using MATLAB bvp4c package.

- Velocity profile is decreasing in M
- A rise in the values of M,K, R and Ec enhances the temperature profiles.
- > Temperature profiles decreases with an increase of Pr, N and Q

- > The concentration profile increases with an increase of M.
- > The concentration profile decreases with an increase of Sc and γ

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