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Study of Multicomponent Cross-Diffusion Systems of Biological Population with Convective Transfer

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Abstract: In this paper investigated properties of cross-diffusion systems of a biological population with double nonlinearity and convective transfer, simulated on computer processes of multicomponent cross-diffusion systems of a biological population of convective transfer, obtained estimates for solving the Cauchy problem of multicomponent cross-diffusion systems of biological population of convective transport.

Keywords. Convective transfer, cross-diffusion, system of equation, biological population.

Introduction

In 1937 Fisher proposed the equation [1; p. 355-360],

$$u_t = ku(1-u) + Du_{xx} \qquad (*)$$

as a deterministic version of the stochastic model of the propagation of a favorable gene in a diploid population, where the scalar function u(t,x) satisfies the given initial and boundary conditions, k and D - positive constants. He examined the equation in detail and obtained a number of useful results. Heuristic and genetically based derivation of the equation was also led by A.N. Kolmogorov, I.G. Petrovsky and N.S. Piskunov, whose classical work served as the basis for a more rigorous analytical approach to the Fisher equation. Fisher's equation is one of the simplest nonlinear equations of reactions with diffusion, in which waves appear [1, p.20-30].
Equation (*) is the simplest differential model for the logistic model of population growth, which gives a solution like a kinematic wave [2; p.33-40, 40; p.30-35].

Consider a reaction-diffusion equation

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial \xi^2} + a\frac{\partial u}{\partial x} + f(u).$$

At $t = t,\ \xi = x + at$, $a = const$ the equation has the form

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial \xi^2} + f(u).$$

In [1; pp. 469-507-136] the existence of global in time weak solutions of the reaction-cross-diffusion systems for an arbitrary number of competing population species has been proved. The equations derived from a lattice random walk model with shared transition rates. In the case of linear transition rates, he expands the population model of two types: Shigezada, Kawasaki and Teramoto. The equations considered in a bounded domain with homogeneous Neumann boundary conditions. The proof of existence based on a refined entropy method and a new approximation scheme. Global existence follows a detailed balance or weak cross-diffusion condition.

Shigesada and others [3; pp. 469-507-136] proposed in their original work a diffuse Lotka-Volterra system for two competing species, which is able to describe the segregation of a population and show the formation of a pattern with increasing time. Based on the lattice random walk model, this system was extended to an arbitrary number of species in [3; pp. 469-507-136]. While the analysis of the existence of weak global solutions for the two-species model is well understood by now, there are only very few results for the n-species model under very restrictive conditions [3; pp. 469-507-136] for the first time, a global analysis of the existence of an arbitrary number of population species using the entropy method is presented, and an amazing relationship is found between the monotonicity of entropy and the detailed state of balance of the associated Markov chain.

At [4; pp.41-67] the system of Maxwell-Stefan equations is considered, which describes multicomponent diffusion flows in undiluted solutions or gas mixtures.

Statement of the task

Definition 1. Unlimitedproblemsolution

$$u_t = ku(1-u) + Du_{xx} \qquad (1)$$

is called localized if for $0 < t < T < +\infty$ exists $0 < L < +\infty$, that $u(t,x) \equiv 0$ at $|x| \in (L, +\infty)$ [5; c.135-136].

Definition 2. Function $u_+(t,x)$ $\left(u_-(t,x)\right)$ is called the upper (lower) solution to the problem

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = \nabla\left(D_1 u_2^{m_1-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_1\right) + k_1 u_1\left(1-u_1^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t} = \nabla\left(D_2 u_1^{m_2-1}\left|\nabla u_1^k\right|^{p-2}\nabla u_2\right) + k_2 u_2\left(1-u_2^{\beta_2}\right), \end{cases}$$

$$u_1\big|_{t=0} = u_{10}(x),\ u_2\big|_{t=0} = u_{20}(x).$$

if satisfies condition at the domain Q [5; c.135-136]

$$L\left(u_+\left(t,x\right)\right) \le 0 \left(L\left(u_-\left(t,x\right)\right) \ge 0\right),$$

$$\text{и}\, u_0\left(x\right) \le u_+(0,x) \left(u_0\left(x\right) \ge u_-(0,x)\right).$$

Definition 3. Solution $u(t,x)$ at every $t \in \left(0,+\infty\right)$ is finite in x if satisfies condition [5; p.135-136]

$$u(t,x) \equiv 0, \quad x \ge l(t) \text{ ,and } u(t,x) > 0 \text{ at} |x| \le l(t), \quad t \in \left(0,+\infty\right).$$

Note that below we will often use solution comparison theorems, which play an important role in the study of nonlinear problems. Having found a particular solution of a self-similar or approximately self-similar equation, then one can use it to compare solutions, which will make it possible, without knowing the solution to the problem, to obtain an estimate of the solutions through a known function, which is very important for the numerical solution of nonlinear

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problems. Therefore, in nonlinear problems, when studying the properties of solutions, an important and sometimes decisive role-played by particular solutions.

Therefore, we will often use, very important from the point of view of applications, the following comparison theorems for solutions [4, pp. 891-905].

Theorem 1. (Solution comparisons). Let $u_1(t,x)\ge 0,\ u_2(t,x)\ge 0,$ nonnegative generalized solutions of equation (1) in $(0,+\infty)\times R^N$, satisfying the conditions [45; p.891-905]

$$u_2(0,x)\ge u_1(0,x),\ x\in R^N,$$

$$\begin{cases} u_2(t,0)\ge u_1(t,0), \\ u_2(t,b)\ge u_1(t,b), \end{cases}$$

$$t\in(0,T),\ (T>0).$$

Then $u_2(t,x)\ge u_1(t,x)$ в $(0,+\infty)\times R^N$.

Note that this theorem requires knowledge of the solutions to the problem, which is generally unknown. In this formulation, the solution comparison theorem is not entirely useful from the point of view of practice. We will also use the following comparison theorem for solutions [4; pp. 135-136].

Theorem 2. Let $D=\{(x,t):\ |x|<l(t),t>0\}$, where function $l(t)\ge 0,t>0$, in the domain $Q=\{(t,x):t>0,x\in R^N\}$ a non-negative generalized solution is defined $u(t,x)$ problem (1) and functions $u_\pm(t,x)\in C^{1,2}_{t,x}\left((0,\infty)\times D\right)\cap C\left([0,\infty)\times \bar{D}\right)$, where $u_\pm(t,x)\ge 0$ continuous functions satisfying, respectively, the inequalities $Lu_+\le 0$, $Lu_-\ge 0$ in $(0,+\infty)\times D$ and

$$u_+(0,x)\ge u_0(x)\ge u_-(0,x),\quad x\in R^N.$$

Then the solution to problem (1.1) satisfies the estimate $u_+(t,x)\ge u(t,x)\ge u_-(t,x)$ in Q.

Function $u_+(t,x)$, $u_-(t,x)$ are respectively called upper and lower solutions of problem (1).

Properties of cross-diffusion systems of a biological population of convective transport

In the domain $\mathrm{Q}=\{(\mathrm{t},\mathrm{x}):\ 0<\mathrm{t},\mathrm{x}\in\mathrm{R}\}$ consider a cross-diffusion system of a biological population:

$$\begin{cases} \dfrac{\partial u_1}{\partial t}=\dfrac{\partial}{\partial x}\left(D_1u_2^{m_1-1}\left|\dfrac{\partial u_1}{\partial x}\right|^{p-2}\dfrac{\partial u_1}{\partial x}\right)+l(t)\dfrac{\partial u_1}{\partial x}+k_1(t)u_1\left(1-u_2^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t}=\dfrac{\partial}{\partial x}\left(D_2u_1^{m_2-1}\left|\dfrac{\partial u_2}{\partial x}\right|^{p-2}\dfrac{\partial u_2}{\partial x}\right)+l(t)\dfrac{\partial u_2}{\partial x}+k_2(t)u_2\left(1-u_1^{\beta_2}\right), \end{cases} \tag{2}$$

$$u_1\big|_{t=0}=u_{10}(x),\ \ u_2\big|_{t=0}=u_{20}(x).$$

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Here: $D_1 u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x}\right|^{p-2}$, $D_2 u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x}\right|^{p-2}$ - diffusion coefficients, $l(t)$- convective transfer rate, $m_1, m_2, p, \beta_1, \beta_2$- positive numeric parameters, $u_1 = u_1(t,x) \geq 0$, $u_2 = u_2(t,x) \geq 0$ solutions of the cross-diffusion system of biological population.

For a qualitative analysis of the system of equations for cross-diffusion of convective transport (2), a self-similar system of equations is constructed.

For this, the method of reference equations and nonlinear splitting was used [1].

To construct systems of the self-similar equation in (4.11), we change the variables:

$$u_1(t,x) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta), \quad \eta = x - \int_0^t l(\zeta)d\zeta,\ u_2(t,x) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta),$$

$$\eta = x - \int_0^t l(\zeta)d\zeta,$$

Change of variables will lead the solution of system (2) to the solution of the following system of equations:

$$\begin{cases} \frac{\partial v_1}{\partial \tau} = \frac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1}\left|\frac{\partial v_1}{\partial \eta}\right|^{p-2}\frac{\partial v_1}{\partial \eta}\right) - k_1(t)e^{[(2-p)k_1+(\beta_1-m_1+1)k_2]t} v_1 v_2^{\beta_1}, \\ \frac{\partial v_2}{\partial \tau} = \frac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1}\left|\frac{\partial v_2}{\partial \eta}\right|^{p-2}\frac{\partial v_2}{\partial \eta}\right) - k_2(t)e^{[(\beta_2-m_2+1)k_1+(2-p)k_2]t} v_1^{\beta_2} v_2, \end{cases} \quad (3)$$

$$v_1\big|_{t=0} = v_{10}(\eta),\ v_2\big|_{t=0} = v_{20}(\eta).$$

In this case, a generalized solution to the problem from the class $u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x}\right|^{p-2} \in C(Q)$, $u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x}\right|^{p-2} \in C(Q)$ and a satisfying system in a generalized sense. System (2) isdegenerateintheregionwhere

$$u_1 = 0, \frac{\partial u_1}{\partial x} = 0, u_2 = 0, \frac{\partial u_2}{\partial x} = 0.$$

The system may not have a classic solution. Therefore, when the equality $(m_1-1)k_2 + (p-2)k_1 = (m_2-1)k_1 + (p-2)k_2$, select the parameter as follows:

$$\tau(t) = \frac{e^{[(m_1-1)k_2+(p-2)k_1]t}}{(m_1-1)k_2+(p-2)k_1} = \frac{e^{[(m_2-1)k_1+(p-2)k_2]t}}{(m_2-1)k_1+(p-2)k_2}.$$

This will lead to the solution of the system of equations:

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$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1} \left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2} \dfrac{\partial v_1}{\partial \eta}\right) - a_1(t)\tau^{b_1} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1} \left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2} \dfrac{\partial v_2}{\partial \eta}\right) - a_2(t)\tau^{b_2} v_1^{\beta_2} v_2. \end{cases} \tag{4}$$

Here:

$$a_1 = k_1\left((p-2)k_1 + (m_1-1)k_2\right)^{b_1},\ b_1 = \frac{(2-p)k_1 + (\beta_1 - m_1 + 1)k_2}{(p-2)k_1 + (m_1-1)k_2},$$

$$a_2 = k_2\left((m_2-1)k_1 + (p-2)k_2\right)^{b_2}, \qquad b_2 = \frac{(\beta_2 - m_2 + 1)k_1 + (2-p)k_2}{(m_2-1)k_1 + (p-2)k_2}.$$

After conditions are met: $b_i = 0$, and $a_i(t) = const$, $i = 1, 2$, we come to the solution of a system of equations of the following form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_1 v_2^{m_1-1} \left|\dfrac{\partial v_1}{\partial \eta}\right|^{p-2} \dfrac{\partial v_1}{\partial \eta}\right) - a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta}\left(D_2 v_1^{m_2-1} \left|\dfrac{\partial v_2}{\partial \eta}\right|^{p-2} \dfrac{\partial v_2}{\partial \eta}\right) - a_2 v_1^{\beta_2} v_2. \end{cases}$$

In order to construct a self-similar system for the cross-diffusion system (4), we initially solve the following system and find its solutions:

$$\begin{cases} \dfrac{d\bar{v}_1}{d\tau} = -a_1 \bar{v}_1 \bar{v}_2^{\beta_1}, \\ \dfrac{d\bar{v}_2}{d\tau} = -a_2 \bar{v}_1^{\beta_2} \bar{v}_2. \end{cases}$$

We are looking for a solution in the form:

$$\bar{v}_1(\tau) = c_1(\tau + T_0)^{-\gamma_1},\ \bar{v}_2(\tau) = c_2(\tau + T_0)^{-\gamma_2},\ T_0 > 0.$$

Here

$$c_1 = 1,\quad \gamma_1 = \frac{1}{\beta_2},\ c_2 = 1,\quad \gamma_2 = \frac{1}{\beta_1}.$$

To find system (4.1), we used the nonlinear splitting method:

$$\begin{aligned} v_1(t,\eta) &= \bar{v}_1(t) w_1(\tau,\eta), \\ v_2(t,\eta) &= \bar{v}_2(t) w_2(\tau,\eta) \quad , \end{aligned} \tag{5}$$

When the equality $\gamma_1(p-2) + \gamma_2(m_1-1) = \gamma_2(p-2) + \gamma_1(m_2-1)$ satisfied, parameter $\tau = \tau(t)$ choose as follows:

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$$\tau_1(\tau)=\int_0^\tau \bar{v}_1^{(p-2)}(t)\bar{v}_2^{(m_1-1)}(t)dt=\begin{cases}\dfrac{1}{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}(T+\tau)^{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}, \\ \qquad\qquad if\ \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)]\neq 0, \\ \ln(T+\tau), \quad if\ \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)]=0, \\ (T+\tau), \quad if\ \ p=2\ \ u\ \ m_1=1.\end{cases}$$

After fulfilling the above conditions regarding the variable $w_i(\tau,x),\ i=1,2$ we obtain a system of quasilinear equations [5]:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_1 w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial\eta}\right|^{p-2}\dfrac{\partial w_1}{\partial\eta}\right)+\psi_1(w_1w_2^{\beta_1}-w_1), \\ \dfrac{\partial w_2}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_2 w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial\eta}\right|^{p-2}\dfrac{\partial w_2}{\partial\eta}\right)+\psi_2(w_2w_1^{\beta_2}-w_2).\end{cases} \tag{6}$$

Here

$$\psi_1=\begin{cases}\dfrac{1}{(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])\tau},\ if\ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)>0, \\ \gamma_1 c_1^{-(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])}, \qquad if\ \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0,\end{cases}$$

$$\psi_2=\begin{cases}\dfrac{1}{(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])\tau}, \quad if\ \ 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]>0, \\ \gamma_2 c_1^{-(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])}, \qquad if\ \ 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]=0.\end{cases} \tag{7}$$

We came from solving system (3) to solving system (7). $\tau\to\infty$ and $\psi_i\to 0$ are solutions of the system of equations asymptotically tends to the solution of the following system of equations [5]:

$$\begin{cases}\dfrac{\partial w_1}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_1 w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial\eta}\right|^{p-2}\dfrac{\partial w_1}{\partial\eta}\right), \\ \dfrac{\partial w_2}{\partial\tau}=\dfrac{\partial}{\partial\eta}\left(D_2 w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial\eta}\right|^{p-2}\dfrac{\partial w_2}{\partial\eta}\right).\end{cases} \tag{8}$$

When building an iterative process choosing initial approximations used this feature. If the following condition is true $1-[\gamma_1(p-2)+\gamma_2(m_1-1)\neq 0$, then the wave solution of system (7) has the following form:

$$w_i(\tau(t),\eta)=f_i(\xi),\ \xi=c\tau\pm\eta,\ \ i=1,2.$$

Here c is the wave velocity, and the solutions of the system $w_i(\tau(t),\eta)=f_i(\xi)$ are found from the following self-similar systems of equations [5]:

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$$\begin{cases} \frac{d}{d\xi}(f_2^{m_1-1}\left|\frac{df_1}{d\xi}\right|^{p-2}\frac{df_1}{d\xi}) + c\frac{df_1}{d\xi} + \mu_1(f_1 - f_1\ f_2^{\beta_1}) = 0, \\ \frac{d}{d\xi}(f_1^{m_2-1}\left|\frac{df_2}{d\xi}\right|^{p-2}\frac{df_2}{d\xi}) + c\frac{df_2}{d\xi} + \mu_2(f_2 - f_2\ f_1^{\beta_2}) = 0. \end{cases} \tag{9}$$

Here:

$$\mu_i = \frac{1}{(1-[\gamma_i(p-2)+\gamma_{3-i}(m_i-1)])},\ i=1,2.$$

Self-similar system of equations has the following localized solution: $\overline{f}_1 = A(a-\xi)_+^{n_1}$,

$$\overline{f}_2 = B(a-\xi)_+^{n_2},$$

$$n_1 = \frac{(p-1)(p-(m_1+1))}{n},\ n_2 = \frac{(p-1)(p-(m_2+1))}{n},$$

$$n = (p-2)^2 - (m_1-1)(m_2-1),$$

when the condition is met:

$$p > 2 + [(m_1-1)(m_2-1)]^{1/2},\ p-(m_i+1) > 0,\ i=1,2,$$

$$\beta_1 = 1/n_1,\ \beta_2 = 1/n_1.$$

Coefficients A and B are the solution to systems of algebraic equations of the following form:

$$(n_1)^{p-1}A^{p-1}B^{m_1-1} = c,$$

$$(n_2)^{p-1}A^{m_2-1}B^{p-1} = c.$$

As:

$$u_1(t,x) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta),\quad u_2(t,x) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta),$$

we get:

$$u_1(t,x) = Ae^{-\int_0^t k_1(\zeta)d\zeta}(c\tau(t)-\xi)_+^{n_1},\ u_2(t,x) = Be^{-\int_0^t k_2(\zeta)d\zeta}(c\tau(t)-\xi)_+^{n_2},\ c>0.$$

Considering:

$$[b\tau(t) - \int_0^t l(\eta)d\eta - x] = 0,$$

when conditions are met

$$x \ge [b\tau(t) - \int_0^t l(\eta)d\eta - x],\ \forall t > 0,$$

get

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$$u_1(t,x) \equiv 0,\ u_2(t,x) \equiv 0, x \geq [b\tau(t) - \int_0^t l(\eta)d\eta - x],\ \forall t > 0.$$

This shows that the localization condition for solving the system of cross-diffusion equations (9) is:

$$\int_0^e l(y)dy < 0, \quad \tau(t) < \infty \text{ for } \forall t > 0. \quad (10)$$

When condition (10) is satisfied, we get a new effect - localization of wave solutions (4.19). If condition (10) is not fulfilled, we obtain a phenomenon called the finite velocity of propagation of the disturbance [5].

In this case $u_i(t,x) \equiv 0$ at $|x| \geq b(t),\ \tau(t) = \int_0^t e^{-(m_1+p-3)\int_0^\zeta k_1(y)dy} d\zeta$, $\tau(t) \to \infty$ at $t \to \infty$.

If the conditions are met $n_1 > 0, n_2 > 0, n > 0$ there is a slow diffusion process. Using the method of nonlinear splitting [1] when finding a solution to equation (4.19), functions of the following form were obtained:

$$\bar{\theta}_1(\xi) = (a-\xi)_+^{n_1},\ \bar{\theta}_2(\xi) = (a-\xi)_+^{n_2},$$

Here: $a > 0, (y)_+ = \max(y, 0),\ \xi < a$.

It is known [1, 2] that for global solutions of the cross-diffusion system (6) to exist, the following inequalities with respect to the function $f(\xi)$:

$$\begin{cases} \dfrac{d}{d\xi}(f_2^{m_1-1}\left|\dfrac{df_1}{d\xi}\right|^{p-2}\dfrac{df_1}{d\xi}) + c\dfrac{df_1}{d\xi} + \mu_1(f_1 - f_1\ f_2^{\beta_1}) \leq 0, \\ \dfrac{d}{d\xi}(f_1^{m_2-1}\left|\dfrac{df_2}{d\xi}\right|^{p-2}\dfrac{df_2}{d\xi}) + c\dfrac{df_2}{d\xi} + \mu_2(f_2 - f_2\ f_1^{\beta_2}) \leq 0, \end{cases}$$

at

$$\beta_1 = 1/n_2,\ \beta_2 = 1/n_1.$$

Consider the functions $\bar{\theta}_1(\xi),\ \bar{\theta}_2(\xi)$, and prove that these functions are the asymptotics of the solutions of system (7) and these solutions are finite [5].

Theorem 3. At $n_1 > 0, n_2 > 0, n > 0$ $\bar{\theta}_1(\xi) = (a-\xi)_+^{n_1},\ \bar{\theta}_2(\xi) = (a-\xi)_+^{n_2}$,
$n_1 = \dfrac{(p-1)(p-(m_1+1))}{n},\ n_2 = \dfrac{(p-1)(p-(m_2+1))}{n}$,
$n = (p-2)^2 - (m_1-1)(m_2-1)$, then at $\xi \to a_-$ a finite solution of system (7) has the asymptotics $f_i(\xi) \sim \bar{\theta}_i(\xi)$.

Proof. The solution to equation (7) is sought in the following form $f_i = \bar{\theta}_i(\xi)y_i(\eta),\ \mathrm{i} = 1,2$. (11)

Here: $\eta = -\ln(a-\xi)$. At $\xi \to a_-$ occurs $\eta \to +\infty$. This allows us to investigate solutions to

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problem (11) for asymptotic stability at $\eta \to +\infty$.

Substituting (7) into (9) with respect to the variables $y_i(\eta)$ we obtain an equation of the following form

$$\frac{d}{d\eta}\left(y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta}-n_iy_i\right|^{p-2}(\frac{dy_i}{d\eta}-n_iy_i)\right)+\left(\frac{e^{-\eta}}{a-e^{-\eta}}-n_i\right)\left(y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta}-n_iy_i\right|^{p-2}(\frac{dy_i}{d\eta}-n_iy_i)\right)+ \tag{12}$$

$$+c\left(\frac{dy_i}{d\eta}-n_iy_i\right)-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}y_i(\eta)(1+e^{-n_{3-i}\beta_i\eta}y_{3-i}{}^{\beta_i})=0.$$

Here the kind of function η defined above.

Solutions of systems of equations (7) and (12) in the interval $[\eta_0,+\infty)$ satisfy the inequalities:

$$y_i(\eta)>0,\ \frac{dy_i}{d\eta}-n_iy_i\neq 0.$$

First, we show that the solutions $y_i(\eta)$ systems, equations (12) have for $\eta\to+\infty$ final limit y_{0i}. For this, we introduce the following notation

$$\omega_i(\eta)=y_{3-i}^{m_i-1}\left|\frac{dy_i}{d\eta}-n_iy_i\right|^{p-2}\left(\frac{dy_i}{d\eta}-n_iy_i\right)$$

$$\omega_i'=-\left(\frac{e^{-\eta}}{a-e^{-\eta}}-n_i\right)\omega_i-c\left(\frac{dy_i}{d\eta}-n_iy_i\right)-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}y_i(\eta)(1+e^{-n_i\beta_i\eta}y_{3-i}{}^{\beta_i}).$$

For further research, we introduce the following auxiliary new function:

$$\phi(\tau,\eta)=-\left(\frac{e^{-\eta}}{a-e^{-\eta}}-n_i\right)\tau-c\left(\frac{dy_i}{d\eta}-n_iy_i\right)-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}y_i(\eta)(1+e^{-n_i\beta_i\eta}y_{3-i}{}^{\beta_i}).$$

Here τ - real numeric parameter. Function $\phi(\tau,\eta)$ does not change sign at some interval $[\eta_1,+\infty)\subset[\eta_0,+\infty)$. For anyone $\eta\in[\eta_1,+\infty)$ the following inequalities hold

$$\omega_i'(\eta)>0,\ \omega_i'(\eta)<0.$$

So the function $\omega_i(\eta)$ has at $\eta\in[\eta_1,+\infty)$ final limit. Given the expression for $\omega_i(\eta)$ we have:

$$\lim_{\eta\to+\infty}\omega_i'(\eta)=\lim_{\eta\to+\infty}\left\{-\left(\frac{e^{-\eta}}{a-e^{-\eta}}-n_i\right)\omega_i-c\left(\frac{dy_i}{d\eta}-n_iy_i\right)-\mu_i\frac{e^{-\eta}}{a-e^{-\eta}}y_i(\eta)(1+e^{-n_i\beta_i\eta}y_{3-i}{}^{\beta_{3-i}})\right\}=0.$$

With considering

$$\xi\to a,\ \lim_{\eta\to+\infty}e^{-\eta}\to 0,\ \lim_{\eta\to+\infty}a-e^{-\eta}\to a,\ \omega_i'=0,$$

we obtain the system of algebraic equations

$\beta_i>1,\ i=1,2:$

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$$(n_1)^{p-1} y_2^{m_1-1} y_1^{p-1} = c,$$
$$(n_2)^{p-1} y_1^{m_2-1} y_2^{p-1} = c.$$

Solving the system of algebraic equations, we obtain $y_i = 1$. Given [5]

(4.21) $f_i(\xi) \sim \bar{\theta}_i(\xi)$.

2) $\beta_i = 1/n_i$, $i = 1,2$. y_i should be a solution of the system [5]

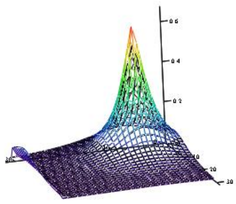
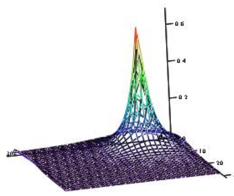
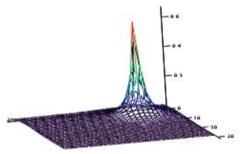
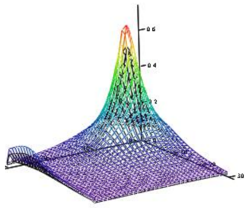
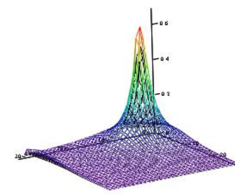
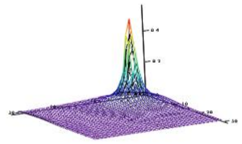
$$(n_1)^{p-1} y_2^{m_1-1} y_1^{p-2} + y_1^{n_1} y_2^{n_1(\beta_1-1)} = c,$$
$$(n_2)^{p-1} y_1^{m_2-1} y_2^{p-2} + y_1^{n_2(\beta_2-1)} y_2^{n_2} = c.$$

Solution to the system of algebraic equations is $y_i = 1$. Considering (4), we get $f_i(\xi) \sim \bar{\theta}_i(\xi)$.

Theorem 3 is proved.

Table 1

Computational Experiment Results. Fast diffusion

Parameter values	$x1=1; x2=1;$ $\lvert x \rvert = \sqrt{2}$	$x1=2; x2=2;$ $\lvert x \rvert = 2\sqrt{2}$	$x1=3; x2=3;$ $\lvert x \rvert = 3\sqrt{2}$
$m_1 = 0.8$, $m_2 = 0.7$, $p = 2.1$ $eps = 10^{-3}$ $\beta_1 = 2$ $\beta_2 = 5$ $m_i + p - 3 < 0$ $n = 3$			
$m_1 = 0.4$, $m_2 = 0.5$, $p = 2.2$ $eps = 10^{-3}$ $\beta_1 = 2$ $\beta_2 = 2$ $m_i + p - 3 < 0$ $n = 5$			

The results of the computational experiment with slow diffusion are shown in Table 1. As an initial approximation, we took u_0, v_0 function $u_0(x,t) = (T+t)^{-\alpha_1}(a - \xi^\gamma)_+^{q_1}$,

$v_0(x,t) = (T+t)^{-\alpha_2}(a - \xi^\gamma)_+^{q_2}$, $\xi = (\int_0^t c(y)dy - x) / \tau^{\frac{1}{p}}$;

$c(t) = 1/(T+t)^n$, $n \geq 1$, $n < 1$, $\int c(y)dy = (T+t)^{1-n} / (1-n)$; $\alpha_1 = \dfrac{1}{\beta_1 - 1}$, $\alpha_2 = \dfrac{1}{\beta_2 - 1}$,

$q_i = \dfrac{(p-1)}{p + m_i - 3}$, $p + m_i - 3 > 0$, $i = 1,2$, $u(x,t) = v(x,t) \equiv 0$

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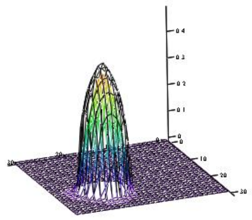
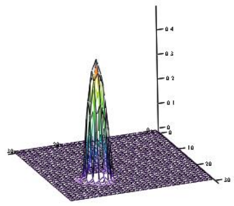
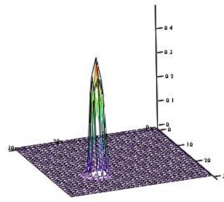
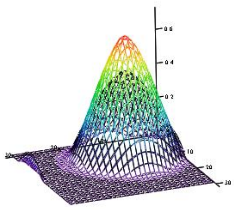
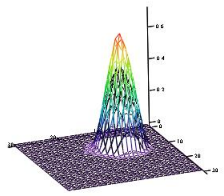
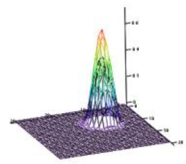
when

$$|x| \geq \int_0^t c(y)dy - a^{(p-1)/p}\tau^{\frac{1}{p}},\ \tau(t) = (T+t)^{1-\alpha_i(m_i+p-3)} / [1-\alpha_i(m_i+p-3)],\ \alpha_i = \frac{\beta_i+1}{\beta_1\beta_2-1},\ i=1,2,\ \beta_1\beta_2>1$$

Table 2

Computational Results for Slow Diffusion

Parameter values	$x1=1;x2=1;$ $\lvert x\rvert=\sqrt{2}$	$x1=2;x2=2;$ $\lvert x\rvert=2\sqrt{2}$	$x1=3;x2=3;$ $\lvert x\rvert=3\sqrt{2}$
$m_1=1.9,\ m_2=5,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $m_i+p-3>0$ $n=3$			
$m_1=1.5,\ m_2=2,\ p=2.5$ $eps=10^{-3}$ $\beta_1=1.5\ \ \beta_2=2$ $m_i+p-3>0$ $n=5$			

Conclusion

In this paper obtained estimates for solving the Cauchy problem for multicomponent cross-diffusion systems of a biological population with double nonlinearity depending on the values of the parameters of the environment and the dimension of space and initial data.

Lower and upper bounds are obtained for the solution of the Cauchy problem by the nonlinear splitting algorithm for the equation of multicomponent cross-diffusion systems of a biological population, which makes it possible to construct the asymptotics of generalized solutions with a compact support and solutions of systems of self-similar equations vanishing at infinity, allowing the problem to be solved numerically.

The problems of choosing initial approximations depending on the values of numerical parameters and data solved, which made it possible to trace the evolution of the reaction-diffusion process.

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