Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

Approximation Algorithm for Optimum Solution of Goal Programming Problem

Monali G. Dhote [1*] , Chandra Prakash Ban [2]
[1]Research Scholar, Himalayan Garhwal University,
Pauri Garhwal - 246001, Uttarakhand, India
[2]Assistant Prof. Dept of Mathematics , Himalayan Garhwal University,
Pauri Garhwal - 246001, Uttarakhand, India.
[*]thakaremonali@gmail.com
[2]cpgban@gmail.com

ABSTRACT

In this paper, an attempt has been made to solve Goal programming problem by using alternative simplex method. These methods will be new approaches and easy to solve goal programming problem. These will be powerful methods to get improved solution. It will take less number of iterations and save valuable time by skipping calculations of net evaluation.

Keywords
Goal Programming Problem, Optimal Solution, proposed algorithm. Less iteration, save time and money.

Introduction

Many times it is not possible to satisfy certain specified goals within given constraints for various problems in management. The problem then turns out to be one among maximizing the degree of attainment of those goals. Goal programming is conscious to clarify this problem of satisfying (possibly conflicting) goals furthermore as probable once a number of them have a higher priority than others.
Linear Programming essentially is that the technique applicable only if there is a single goal (objective function), like maximizing the profit or minimizing the price or loss. There are conditions wherever the system might have multiple (possibly conflicting) goals. As an example, a firm might have a collection of goals, like employment stability, high product quality, maximization of profit, minimizing overtime or price, etc. in such conditions; we want a diverse technique that looks for a compromise solution supported on the relative importance of every objective. This technique is recognized as Goal Programming. It plans to minimizing the variations from the targets that were place by the management.
It's basic approach is to determine a exact numeric goal for each of the objectives, formulate an objective function for each objective, then seek a answer that minimizes the (weighted) sum of deviations of those objective functions from their respective goals.
In 1961, Charnes and Cooper studied Management Models and Industrial Applications of Linear Programming. Charnes et al. (1968) discussed a goal programming model for media planning. Contini (1968) studied a stochastic approach to goal programming. Dauer and Krueger (1977) developed an Iterative Approach to Goal Programming. Kornbluth and Steuer (1981) calculated Goal programming with linear fractional criteria. Moitra and Pal (2002)

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

discussed a fuzzy goal programming approach for solving bilevel programming problems. Pramanik and Kumar (2006) applied Fuzzy goal programming approach to multi- level programming problems. Baky (2009) developed Fuzzy goal programming algorithm to solve decentralized bi-level multi-objective programming problems. Khobragade; Vaidya and. Lamba. (2014) investigated an Approximation algorithm for optimal solution to the linear programming problem. Birla et al. (2017) developed An Alternative Approach for Solving Bi-Level Programming Problems. Putta Baburao and Khobragade (2019) given derivation of Optimum solution of Goal and Fractional Programming Problem.
In this paper, an alternative method has been suggested and solved goal programming problem (GPP).

PROPOSED ALGORITHM FOR GOAL PROGRAMMING PROBLEM

Here we added the following steps of alternative method to solve Goal Programming Problems.

Step (1). Choose $\min \sum x_{ij}, \quad x_{ij} \geq 0$, for entering vector.
Step (2). Choose highest coefficient of decision variables.

(a) If highest coefficient is unique, then element corresponding to this row and column becomes pivotal element.
(b) If highest coefficient is not unique, then apply tie breaking technique.

Step (3). Ignore corresponding row and column. Proceed to step 2 for remaining elements and repeat the same procedure until an optimal solution is obtained or there is an indication for unbounded solution.
Step (4). If all rows and columns are ignored, then optimal solution exists.

Statement of the Problems

Solve the following GPP.

Ex 1. A production manager found the problem of job allocation among two assembly lines. The rate of production of assembly line 1 is 10 units per hour, and that of assembly line 2 is 12 units per hour. The normal working slot for both lines is 8 hours per day. The production manager has situated the following targets for the next day, scheduled as per their importance:

i) Avoid any underachievement of the production level, which is set at 200 units of product.
ii) Avoid any overtime operation of line 2 beyond four hours.
iii) Avoid any underutilization of regular working hours (assign differential weights according to the relative productivity of the two lines).
iv) Minimize overtime in both assembly lines.

Formulate using proposed algorithm and solve.

Solution

Let $x_1 \, and \, x_2$ denotes production rate of assembly line 1 and 2 respectively.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

Then, the constraints and goals of the problem can be expressed as follow:

$$Maximize\ z = 10x_1 + 12x_2$$

$$x_2 \le 12, \qquad x_1 \le 8$$

The problem can now be formulated as goal programming model as follows:

$$Minimize\ Z = p_1 d_1^- + p_2 d_2^+ + p_3(10d_4^- + 12d_2^-) + p_4(d_2^+ + d_4^+)$$

Subject to the constraints: $10x_1 + 12x_2 + d_1^- - d_1^+ = 200$

$$x_2 + d_2^- + d_3^- - d_3^+ = 12$$

$$x_1 + d_4^- - d_4^+ = 8$$

$$x_1,\ x_2,\ d_1^+,\ d_1^-,\ d_2^-,\ d_3^-,\ d_3^+,\ d_4^-,\ d_4^+ \ge 0$$

Step (1). Initial table:

			0	0	p_1	$12p_3$	0	$10p_3$	0	p_2+p_4	0	p_4
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
p_1	d_1^-	200	**10**	12	1	0	0	0	-1	0	0	0
0	d_3^-	12	0	1	0	1	1	0	0	0	-1	0
$10p_3$	d_4^-	8	1	0	0	0	0	1	0	0	0	-1
			↑		↓							

Since $\min \sum x_{ij} = 11$

Hence the column vector x_1 enter in the basis and the column vector d_1^- leaves the basis.

Step (2): Introduce x_1 and drop d_1^-

			0	0	p_1	$12p_3$	0	$10p_3$	0	p_2+p_4	0	p_4
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	20	1	12/10	1/10	0	0	0	-1/10	0	0	0
0	d_3^-	12	0	**1**	0	1	1	0	0	0	-1	0
$10p_3$	d_4^-	-12	0	-12/10	-1/10	0	0	1	-1/10	0	0	-1
				↑			↓					

Since $\min \sum x_{ij} = 1$

Hence the column vector x_2 enter in the basis and the column vector d_3^- leaves the basis.

Step (3): Introduce x_2 and drop d_3^-

			0	0	p_1	$12p_3$	0	$10p_3$	0	p_2+p_4	0	p_4
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

0	x_1	56/10	1	0	1/10	-12/10	-12/10	0	-1/10	0	12/10	0
0	x_2	12	0	1	0	1	1	0	0	0	1	0
$10p_3$	d_4^-	24/10	0	0	-1/10	12/10	12/10	1	**1/10**	0	-12/10	1
								↓	↑			

Since $\min \sum x_{ij} = 0$

Hence the column vector d_1^+ enter in the basis and the column vector d_4^- leaves the basis.

Step (4): Introduce d_1^+ and drop d_4^-

			0	0	p_1	$12p_3$	0	$10p_3$	0	p_2+p_4	0	p_4
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	8	1	0	0	0	0	1	0	0	0	-1
0	x_2	12	0	1	0	1	1	0	0	0	1	0
$10p_3$	d_1^+	24	0	0	-1	12	12	10	1	0	-12	-10

Optimum solution is

$$x_1 = 8,\ x_2 = 12,\ d_1^+ = 24,\ d_1^- = d_2^- = d_3^- = d_4^- = 0$$

Ex 2. A rural clinic appoints its staff from cities and towns nearby it on a clock-hour basis. The cities effort to include a general practitioner (GP), a nurse and an internist on job for some days of each week. The clinic has a weekly expense of Rs. 1,200. A GP charges the clinic Rs. 40 per hour, a nurse charges Rs. 20 per hour, and an internist charges Rs. 150. The clinic has made the following goals as per priority:

i) A nurse should be present for not more than 30 hours per week.
ii) The weekly budget of Rs. 1,200 should not be exceeded.
iii) A GP and internist should be present for not more than 20 hours per week.
iv) An internist should be available at least 6 hours per week.

Formulate using proposed algorithm and solve.

Solution

Let x_1, x_2 *and* x_3 denotes charges of GP, nurses and internist respectively.

Then, the constraints and goals of the problem can be expressed as follow:

$$Maximize\ Z = 40x_1 + 20x_2 + 150x_3$$

$$x_2 \le 30, \qquad x_1 + x_3 \le 20,\ x_3 \le 6$$

The problem can now be formulated as goal programming model as follows:

$$Minimize\ Z = p_1 d_1^- + p_2 d_2^+ + p_3 d_3^- + p_4 - d_4^-$$

Subject to the constraints: $x_2 + d_1^- - d_1^+ = 30$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

$$40x_1 + 20x_2 + 150x_3 + d_2^- - d_2^+ = 1200$$
$$x_1 + x_3 + d_3^- - d_3^+ = 20$$
$$x_3 + d_4^- - d_4^+ = 6$$
$$x_1,\ x_2,\ d_1^+,\ d_1^-,\ d_2^-, d_2^-,\ d_3^-,\ d_3^+,\ d_4^-,\ d_4^+ \geq 0$$

Step (1): Initial table:

			0	0	0	p_1	0	p_3	p_4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
p_1	d_1^-	30	0	1	0	1	0	0	0	1	0	0	0
0	d_2^-	1200	40	**20**	150	0	1	0	0	0	1	0	0
p_3	d_3^-	20	1	0	1	0	0	1	0	0	0	1	0
p_4	d_4^-	6	0	0	1	0	0	0	1	0	0	0	1

↑ ↓

Since $\min \sum x_{ij} = 21$

Hence the column vector x_2 enter in the basis and the column vector d_2^- leaves the basis.

Step (2): Introduce x_2 and drop d_2^-

			0	0	0	p_1	0	p_3	p_4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
p_1	d_1^-	-30	-2	0	-15/2	1	-1	0	0	1	-1	0	0
0	x_2	60	2	1	15/2	0	1/20	0	0	0	1/20	0	0
p_3	d_3^-	20	**1**	0	1	0	0	1	0	0	0	1	0
p_4	d_4^-	6	0	0	1	0	0	0	1	0	0	0	1

↑ ↓

Since $\min \sum x_{ij} = 1$

Hence the column vector x_1 enter in the basis and the column vector d_3^- leaves the basis.

Step (3): Introduce x_1 and drop d_3^-

			0	0	0	p_1	0	p_3	p_4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
p_1	d_1^-	10	0	0	-11/2	1	-1	2	0	1	-1	2	0
0	x_2	20	0	1	11/2	0	1/20	-2	0	0	1/20	-2	0
0	x_1	20	1	0	1	0	0	1	0	0	0	1	0
p_4	d_4^-	6	0	0	**1**	0	0	0	1	0	0	0	1

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

↑ ↓

Since $\min \sum x_{ij} = 2$

Hence the column vector x_3 enter in the basis and the column vector d_4^- leaves the basis.

Step (4): Introduce x_3 and drop d_4^-

			0	0	0	p_1	0	p_3	p_4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
p_1	d_1^-	43	0	0	0	1	-1	2	11/2	**1**	-1	2	11/2
0	x_2	13	0	1	0	0	1/20	-2	-11/2	0	1/20	-2	-11/2
0	x_1	14	1	0	0	0	0	1	-1	0	0	1	-1
0	x_3	6	0	0	1	0	0	0	1	0	0	0	1

↓ ↑

Since $\min \sum x_{ij} = 1$

Hence the column vector d_1^+ enter in the basis and the column vector d_1^- leaves the basis.

Step (5): Introduce d_1^+ and drop d_1^-

			0	0	0	p_1	0	p_3	p_4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	d_1^+	43	0	0	0	1	-1	2	11/2	1	-1	2	11/2
0	x_2	13	0	1	0	0	1/20	-2	-11/2	0	1/20	-2	-11/2
0	x_1	14	1	0	0	0	0	1	-1	0	0	1	-1
0	x_3	6	0	0	1	0	0	0	1	0	0	0	1

Optimum solution is

$$x_1 = 14,\ x_2 = 13,\ x_3 = 6,\ d_1^+ = 43,\ d_1^- = d_2^- = d_3^- = d_4^- = 0.$$

Ex 3. A firm manufactures two products, says P and Q. Product P sells for a net profit of Rs. 80 per unit and product Q sells for a net profit of Rs. 40 per unit. The firm wants to earn Rs. 900 in the next week. Also, the management would like to attain sales volume for the two products near to 17 and 15 respectively. Formulate using proposed algorithm and solve.

Solution

Let x_1 *and* x_2 be the number of units of product P and Q respectively.

Then, the linear programming formulation of the problem is

$$Maximize\ Z = 80x_1 + 40x_2$$

$$x_1 \le 17\ ,\ x_2 \le 15\ \ and\ \ x_1 \ge 0\ ,\ x_2 \ge 0$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

The problem can now be formulated as goal programming model as follows:

$$Minimize\ Z = d_1^- + d_1^+ + d_2^- + d_3^-$$

Subject to the constraints: $80x_1 + 40x_2 + d_1^- - d_1^+ = 900$

$$x_1 + d_2^- = 17$$

$$x_2 + d_3^- = 15$$

$$x_1,\ x_2,\ d_1^+, d_1^-, d_2^-, d_3^- \geq 0$$

Step (1): Initial table:

			0	0	1	1	1	1
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
1	d_1^-	900	80	**40**	1	0	0	-1
1	d_2^-	17	1	0	0	1	0	0
1	d_3^-	15	0	1	0	0	1	0

↑ ↓

Since $\min \sum x_{ij} = 40$

Hence the column vector x_2 enter in the basis and the column vector d_1^- leaves the basis

Step (2): **Introduce x_2 and drop d_1^-**

			0	0	1	1	1	1
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
0	x_2	45/2	2	1	1/40	0	0	-1/40
1	d_2^-	17	**1**	0	0	1	0	0
1	d_3^-	-15/2	-2	0	-1/40	0	1	1/40

↑ ↓

Since $\min \sum x_{ij} = 1$

Hence the column vector x_1 enter in the basis and the column vector d_2^- leaves the basis

Step (3): Introduce x_1 and drop d_2^-

			0	0	1	1	1	1
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
0	x_2	-23/2	0	1	1/40	-2	0	-1/40
0	x_1	17	1	0	0	1	0	0
1	d_3^-	53/2	0	0	-1/40	-2	1	**1/40**

↓ ↑

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

Since $\min \sum x_{ij} = 0$

Hence the column vector d_1^+ enter in the basis and the column vector d_3^- leaves the basis

Step (4): Introduce d_1^+ and drop d_3^-

			0	0	1	1	1	1
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_1^+
0	x_2	15	0	1	0	-4	1	0
0	x_1	17	1	0	0	1	0	0
1	d_1^+	1060	0	0	-1	-80	40	1

Optimum solution is

$$x_1 = 17,\ x_2 = 15,\quad d_1^+ = 1060,\ d_1^- = d_2^- = d_3^- = 0$$

Ex 4. Delton Electronics manufactures two sorts of TV sets. One TV set, the deluxe needs 2 hours in assembly, while the other, the Supreme requires 4 hours assembly time. The regular assembly operation is limited to 80 hours per week. Surveys of marketing shows that exactly 60 Deluxe and 30 Supreme TV sets should be produced every week. The net profit from the deluxe model is Rs. 100 each and Rs.150 each from the Supreme model.

The company president has started the following objectives in order to priority.

- Maximize total profit.
- Minimize overtime operation of the assembly line.
- Sell as many TV sets as possible (this is not necessarily the same as maximizing profit). Since the net profit from the Supreme model is 2 times that from the Deluxe model, the president has 2 times as much as desire to maximize the sales of the Supreme model as he does for the Deluxe model.

Formulate using proposed algorithm and solve.

Solution

Let x_1 *and* x_2 denotes the number of Deluxe TV sets and number of Supreme TV sets respectively.

Then, the linear programming formulation of the problem is:

$$Maximize\ Z = 100x_1 + 150x_2$$

$$2x_1 + 4x_2 \le 80\ ,\ x_1 \le 60,\ \ x_2 \le 30$$

The problem can now be formulated as goal programming model as follows:

$$Minimize\ Z = d_1^- + d_2^+ + (d_3^- + d_4^-) + (d_3^- + 2d_4^-)$$

Subject to the constraints: $100x_1 + 150x_2 + d_1^- - d_1^+ = 5000$

$$2x_1 + 4x_2 + d_2^- - d_2^+ = 80$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

$$x_1 + d_3^- - d_3^+ = 60$$
$$x_2 + d_4^- - d_4^+ = 30$$
$$x_1,\ x_2,\ d_1^-, d_2^-, d_3^-, d_4^-, d_1^+, d_2^+, d_3^+, d_4^+, \geq 0$$

Step (1): Initial table:

			0	0	1	0	2	3	0	1	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
1	d_1^-	5000	**100**	150	1	0	0	0	-1	0	0	0
0	d_2^-	80	2	4	0	1	0	0	0	-1	0	0
2	d_3^-	60	1	0	0	0	1	0	0	0	-1	0
3	d_4^-	30	0	1	0	0	0	1	0	0	0	-1

Since $\min \sum x_{ij} = 103$

Hence the column vector x_1 enter in the basis and the column vector d_1^- leaves the basis

Step (2): **Introduce x_1 and drop d_1^-**

			0	0	1	0	2	3	0	1	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	50	1	3/2	1/100	0	0	0	-1/100	0	0	0
0	d_2^-	-20	0	**1**	-1/50	1	0	0	1/50	-1	0	0
2	d_3^-	10	0	-3/2	-1/100	0	1	0	1	0	-1	0
3	d_4^-	30	0	1	0	0	0	1	0	0	0	-1

Since $\min \sum x_{ij} = 2$

Hence the column vector x_2 enter in the basis and the column vector d_2^- leaves the basis

Step (3): Introduce x_2 and drop d_2^-

			0	0	1	0	2	3	0	1	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	80	1	0	-1/50	-3/2	0	0	-1/25	3/2	0	0
0	x_2	-21	0	1	-1/50	1	0	0	1/50	-1	0	0
2	d_3^-	-20	0	0	-1/25	3/2	1	0	103/100	-3/2	-1	0
3	d_4^-	50	0	0	1/50	-1	0	1	-1/50	**1**	0	-1

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

↓ ↑

Since $\min \sum x_{ij} = 0$

Hence the column vector d_2^+ enter in the basis and the column vector d_4^- leaves the basis

Step (4): Introduce d_2^+ and drop d_4^-

			0	0	1	0	2	3	0	1	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	5	1	0	1/100	0	0	-3/2	- 1/100	0	0	3/2
0	x_2	30	0	1	0	0	0	1	0	0	0	-1
2	d_3^-	55	0	0	- 1/100	0	1	3/2	**1**	0	-1	-3/2
1	d_2^+	50	0	0	1/50	−1	0	1	-1/50	1	0	-1

↑ ↓

Since $\min \sum x_{ij} = 0.97$

Hence the column vector d_1^+ enter in the basis and the column vector d_3^- leaves the basis

Step (5): Introduce d_1^+ and drop d_3^-

			0	0	1	0	2	3	0	1	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_2^-	d_3^-	d_4^-	d_1^+	d_2^+	d_3^+	d_4^+
0	x_1	55.5	1	0	999/1000	0	1/ 100	- 297/200	0	0	- 1/100	303/200
0	x_2	30	0	1	0	0	0	1	0	0	0	-1
0	d_1^+	55	0	0	-1/100	0	1	3/2	1	0	-1	-3/2
1	d_2^+	51.1	0	0	99/5000	-1	1	76/500	0	1	-1/50	-76/50

Optimum solution is

$$x_1 = 55.5,\ x_2 = 30,\quad d_1^+ = 55,\ d_2^+ = 51.1,\ d_1^- = d_2^- = d_3^- = d_4^- = 0$$

Conclusion

In this article, an alternative simplex method for Goal programming problem has been suggested. It is observed that the proposed method reduces number of iterations, saves valuable time and got optimum solutions. Therefore, our method is most powerful method and provides results in lesser time.

References

[1] Birla Rashmi, Agarwal Vijay K., Khan Idrees A., Vishnu Narayan Mishra (2017), An

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 10581 - 10591
Received 25 April 2021; Accepted 08 May 2021.

Alternative Approach for Solving Bi-Level Programming Problems, *American Journal of Operations Research*, 7, pp 239-247, http://www.scirp.org/journal/ajor

[2] Charnes A. and Cooper W. W.(1961), *Management Models and Industrial Applications of Linear Programming*, Vol. I. John Wiley, New York.

[3] Charnes A., Cooper W. W., Devoe J. K., Learner D. B. And Reinecke W.(1968), *"A goal programming model for media planning". Mgmt Sci.* 14 , pp. 423-430.

[4] Contini B.(1968), A stochastic approach to goal programming. *Ops Res*. 16, pp 576-586.

[5] Dauer Jerald P. And Krueger Robert J.(1977), *"An Iterative Approach to Goal Programming", Operation Research Q., Vol.* 28, 3, *pp*. 671-681.

[6] Ibrahim A. Baky (2009), Fuzzy goal programming algorithm for solving decentralized bi-level multi- objective programming problems , *Fuzzy Sets and Systems* 160, pp 2701–2713.

[7] Khobragade N. W; Vaidya N. V. and Navneet K. Lamba.(2014), Approximation algorithm for optimal solution to the linear programming problem, *Int. J. Mathematics in Operational Research,* Vol. 6, No. 2, pp 139-154.

[8] Kornbluth J. S. H., Steuer R.E.(1981), Goal programming with linear fractional criteria, *European Journal of Operational Research* 8, pp 58–65.

[9] Moitra B. N., Pal B. B.(2002), A fuzzy goal programming approach for solving bilevel programming problems, in: N.R. Pal, M. Sugeno (Eds.), AFSS 2002, *Lecture Notes in Artificial Intelligence,* Vol. 2275, Springer, Berlin, Heidelberg, pp. 91–98.

[10] Pramanik S., Kumar Roy T.(2006), Fuzzy goal programming approach to multi-level programming problems, *European Journal of Operational Research* 176, pp 1151–1166.

[11] Putta Baburao and Khobragade N. W.(2019), "Optimum solution of Goal and Fractional Programming Problem" , *Int. J of Management, Technology and Engg*, Vol IX, Issue IX, pp 142-152.