

# Novel Mathematical Model for Traffic Signal System Using Adaptive Queue Length Method

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## ABSTRACT

In the present day traffic signal system, the time cycle is fixed which causes congestion most of times. Research works carried out in this subject area so far focused on finding good estimate for the delay experienced by vehicles. In this paper, a novel technique based on adaptive queue length is proposed that will help to dynamically switch the traffic depending on the traffic situation and helps to reduce the congestions and ensures smooth traffic flow. Congestion problems at the intersection point can be minimized using the proposed method in which traffic signals can adaptively switch traffic lights according to the prevailing scenario. Mathematical analysis is carried out to determine the number of vehicles arrive at the junction which will be used for switching of traffic signals according to the traffic. For the determination of arrival rate of vehicle, Poisson's distribution is used and mathematical equations are derived in this research work. The duration for which green light should alter/change exactly according to the number of vehicles present at the intersection has been derived from the mathematical analysis proposed in this method. To justify the mathematical analysis, simulation of heavy load scenario is done using software tools, and results shows that the proposed algorithm works efficiently and minimize the delay as compared to the pre-timed traffic signal. The proposed technique, by dynamically switching the traffic according to the prevailing traffic situation, helps to eliminate traffic congestion and delay to a large extent.

## KEYWORDS

Adaptive Traffic Signal Control, Adaptive Queue Length, Fixed-cycle Traffic-light, Poisson's Distribution, VanetModel.

## Introduction

As number of vehicles are increasing, traffic congestion is witnessed very frequently now a days. In the present system of fixed time duration traffic signals, the signal cycle is fixed and continues to be the same even after several years of its commissioning. The current traffic signal system is designed with pre-determined signal times which are arrived at based on the traffic load estimation. But, with the exponential growth in the number of vehicles, these traffic load calculations and the signal timings often go wrong and the traffic signals are unable to cope up with the present scenario which causes the traffic congestions everywhere. In this research work, dynamic control of traffic signal system is proposed to resolve the traffic congestion problem. In the proposed method, depending on the traffic situation, the traffic signal dynamically changes and the time duration of signals is designed using a novel mathematical model based on adaptive queue length method.

## Literature Survey

One of the main considerations in Vehicular Adhoc networks (VANET) is to create a network with different moving vehicles and connecting devices, with objective to reduce the latency. In the current scenario where number of vehicles is increasing every day, intelligent transportation system is required with adaptive switching of traffic on the basis of the queue length of the vehicles. D.T. Dissanayake et.al., [3] proposed a system to detect moving metal objects (vehicles) and then to calculate the appropriate duration for the traffic signals at an intersection to operate. By employing mathematical functions to calculate the appropriate timing for the green signal to illuminate, the authors have proposed to solve the problem of traffic congestion. Wireless sensor network based traffic management system

was proposed by Chen Wenjie et.al., [9] to detect the vehicles and to monitor their movement. However, the disadvantage in this system is that it can detect the vehicle only from a fixed position. The authors in [6] proposed an algorithm for traffic signal control using Vehicular adhocnetwork based system. It has the advantage of eliminating the time when no vehicle passing across. Authors in [1] proposed a system which is adaptive to traffic flow and reduces the average waiting time of vehicle at traffic signals.

Asif Ahmad et.al., [7] focused on a simple 4-way junction as well as complex system having lanes in more than 4 directions. They have proposed two algorithms, i.e. the earliest deadline first (EDF) and the fixed priority (FP). These algorithms work efficiently in complex road scenario and heavy traffic load. Authors in [5] proposed adaptive signal flow system by deploying sensors in each lane of the road and central controller collects the information and processes. KamonthepTiaprasert et.al, [2] proposed an algorithm for queue length estimation using connected vehicle technology. Authors in [4] have proposed VANET based intelligent transport system as a solution to traffic congestion. J.S.H. Van Leeuwaarden, [8] proposed in his research article, to determination of arrival rate of vehicle using delay analysis by making some assumptions on discrete-time arrival of vehicle.

## Proposed Work

### Proposed Technique for Traffic Control System based on Adaptive Queue Length

A novel adaptive queue length based method is proposed in this work, to minimize the traffic congestion. In this method, traffic signals are switched/changed adaptively by assessing the present traffic situation. In the normal traffic system, fixed time is allocated to each traffic signal irrespective of the size of the traffic. This system causes congestion during peak traffic times and traffic delays occur due to fixed signal timing. As the number of vehicles and the traffic increases, more congestion is experienced on roads.

In this proposed work, the arrival rate of vehicle is considered to be variable at all times in all traffic roads. Matlab simulation software is used to determine the arrival rate of the vehicle, based on which traffic signals are switched. In the conventional traffic system, variable arrival time of vehicle causes traffic congestion as the signals are of fixed cycle type. In the proposed technique, traffic signals are switched with different time cycles based on the number of vehicles arriving, which resolves the traffic jam problems. In this method, the timing cycle of each traffic signal either increase or decrease depending on the traffic situation. The proposed method works in the traffic scenario as depicted in Fig. 1.

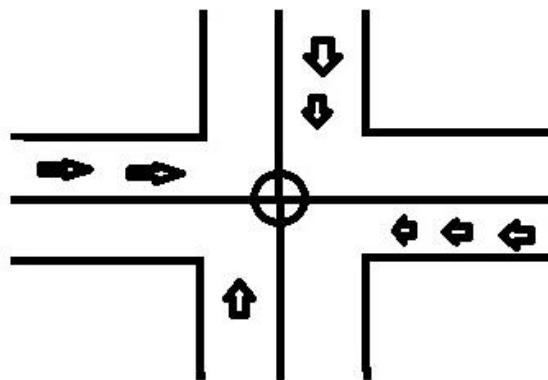


Fig. 1. Scenario for Proposed Method

### Mathematical Model

The assumptions made for the mathematical model in the proposed method are enumerated below.

(i) Discrete time is assumed for different traffic signals. If, 'c' is the total cycle time and 'g' and 'r' are the time slots for green and red signals, then;

$$c = g + r \quad (1)$$

(ii) The no. of vehicles  $A_{k,n}$ , time slot 'k' and cycle 'n', all are independent of each other and are identically

distributed.

(iii)Poisson’s distribution is used to determine the vehicle arrival rate at the traffic junction.

As per the above assumptions, the mathematical model has been derived to determine the duration of each traffic signal depending on the number of vehicles arriving at the junction.The traffic delays are reduced using this model when compared to the conventional traffic signal system. Using this model, the traffic signals (G-Green and R-Red) are switched as shown in Fig. 2.



**Fig. 2.** Adaptive traffic cycle duration

Consider  $Q_{Gn}^k$  as the initial queue length at the intersection with ‘n’ as the identification of the lane and the threshold value  $G_t$  is the maximum no. of vehicles that can be served in a lane in one cycle.Further,  $A_{k+1}$  is taken as the new vehicle arrival at the junction before/after the completion of one cycle and  $\alpha$  &  $\beta$  be the number of vehicles served in one cycle. For the determination of arrival rate of vehicle Poisson distribution is used.

For Path 1 (p1):

$$Q_{G1}^{k+1} = \begin{cases} Q_{G1}^k + A_{k+1} - \alpha_1; & Q_{G1}^k > 0 \\ A_{k+1}; & Q_{G1}^k = 0 \end{cases} \quad (2)$$

Where,  $\alpha_1 = \min\{Q_{G1}^k, G_t\}$  and  $G_t$  is the Threshold value.

For Path 2 (p2):

$$Q_{G2}^{k+1} = \begin{cases} Q_{G2}^k + A_{k+1} - \alpha_2; & Q_{G2}^k > 0 \\ A_{k+1}; & Q_{G2}^k = 0 \end{cases} \quad (3)$$

Where,  $\alpha_2 = \min\{Q_{G2}^k, G_t\}$

For Path 3 (p3):

$$Q_{G3}^{k+1} = \begin{cases} Q_{G3}^k + A_{k+1} - \beta_1; & Q_{G3}^k > 0 \\ A_{k+1}; & Q_{G3}^k = 0 \end{cases} \quad (4)$$

Where,  $\beta_1 = \min\{Q_{G3}^k, G_t\}$

For Path 4 (p4):

$$Q_{G4}^{k+1} = \begin{cases} Q_{G4}^k + A_{k+1} - \beta_2; & Q_{G4}^k > 0 \\ A_{k+1}; & Q_{G4}^k = 0 \end{cases} \quad (5)$$

Where,  $\beta_2 = \min\{Q_{G4}^k, G_t\}$

From Eq. (2) to (5), the number of arrived vehicles at the intersection can be evaluated. For the calculation of time cycle for green signal,  $\alpha$ ,  $\beta$  and  $\gamma$  are to be evaluated, as given below:

From path 1 and path 2, it can be derived that,

$$\alpha = \text{Max}\{\alpha_1, \alpha_2\} \quad (6)$$

Similarly, from path 3 and path 4 we get,

$$\beta = \text{max}\{\beta_1, \beta_2\} \quad (7)$$

Thus, we get

$$\gamma = \text{max}\{\alpha, \beta\} \quad (8)$$

Where,  $\gamma$  is final time cycle for the green signal for vehicle to be served. Similarly, time cycle for red signal will be the minimum value of  $\alpha$  and  $\beta$ .

Now from the probability theory we are calculating pdf at random queue length as j.

Probability for p1 at queue length j and  $x > \alpha_1$ ,

$$P\{Q_{G1}^{k+1} = j\} = \sum_{x=\alpha_1+1}^{j+\alpha_1} P\{Q_{G1}^k = x\} P\{A_{k+1} = j - x + \alpha_1\} \forall j \geq 1 \quad (9)$$

Probability for p1 at queue length j and  $x < \alpha_1$

$$P \{Q_{G_1}^{k+1} = j\} = \sum_{x=0}^{\alpha_1-1} P \{Q_{G_1}^k = x\} P\{A_{k+1} = j\} \quad (10)$$

Probability for p1 at queue length j and  $x \leq \alpha_1$

$$P \{Q_{G_1}^{k+1} = 0\} = \sum_{x=0}^{\alpha_1} P \{Q_{G_1}^k = x\} P\{A_{k+1} = 0\} \quad (11)$$

From Eq. (11), it can be derived that the probability at initial queue length ( $P \{Q_{G_1}^k = x\}$ ) is dependent on initial vehicles' queue length and here the number of arriving vehicles are zero ( $P\{A_{k+1} = 0\}$ ).

## Results

Random number of vehicles are generated using Matlab software tool with the help of exponential distribution in four lanes by fixing a threshold for green signal time cycle. Using the above mathematical analysis with the help of Matlab code, the final time cycle for green light and red light are calculated. Fig. 3 and 4 shows the arrival rate of vehicle at the junction, threshold value of the green light duration and the other data at the end of each cycle. It is considered that the length of each vehicle is 10 units which take 1 second to cross the intersection. Fig. 5 shows the number of vehicles arrived on each lane at the junction. With these data values, the traffic signals are dynamically changed and the duration of the green light and red light are switched/changes adaptively according to the arrival rate of the vehicles at the junction.

Name	Value	Min	Max
A	50	50	50
ArrivalTime	[10001,10018,10182,1...	10001	13104
AverageArrivalTime	[100,10,500,1000]	10	1000
AveragePacketLen...	50	50	50
B	50	50	50
Buffer_Threshold	1000	1000	1000
C	14	14	14
CurBufferSize	[970,990,140,120]	120	990
D	12	12	12
G	[50,50,50,50]	50	50
M	4	4	4
PacketBuff	<4x500 double>	0	99
PacketLength	[10,10,10,10]	10	10
SlotTime	2	2	2
TotalTime	10000	10000	10000
TotalTimeInSeconds	20000	20000	20000
U	-36	-36	-36
V	-38	-38	-38
X	47	47	47
Y	49	49	49
a	97	97	97
alpha	50	50	50
alpha1	49	49	49
b	99	99	99
beta	14	14	14
beta1	-36	-36	-36
c	14	14	14
d	12	12	12
green1	50	50	50

Fig. 3. Output data of Mathematical analysis-I

Name	Value	Min	Max
PacketLength	[10,10,10,10]	10	10
SlotTime	2	2	2
TotalTime	10000	10000	10000
TotalTimeInSeconds	20000	20000	20000
U	-36	-36	-36
V	-38	-38	-38
X	47	47	47
Y	49	49	49
a	97	97	97
alpha	50	50	50
alpha1	49	49	49
b	99	99	99
beta	14	14	14
beta1	-36	-36	-36
c	14	14	14
d	12	12	12
green1	50	50	50
green2	49	49	49
i	4	4	4
j	4	4	4
k	4	4	4
red1	14	14	14
red2	0	0	0
t	10000	10000	10000
th	50	50	50
u	-36	-36	-36
v	-38	-38	-38
x	47	47	47
y	49	49	49

Fig. 4. Output data of Mathematical analysis-II

	1	2	3	4	5	6	7	8
1	97	10	10	10	10	10	10	10
2	99	10	10	10	10	10	10	10
3	14	10	10	10	10	10	10	10
4	12	10	10	10	10	10	10	10

Fig. 5. Number of vehicles arrived at junction

## Conclusion

In this work, an algorithm is proposed using which the traffic signal can be dynamically changed as per prevailing traffic scenario and thus it helps to minimize the delay experienced by the vehicles in heavy traffic scenario. Detailed mathematical analysis is provided in this paper and equations are derived using which, the time cycles of green and red signals can be evaluated which will help in switching of traffic signals dynamically. To justify the mathematical analysis, simulation of heavy load scenario is done using the Matlab software tool, and results show that the proposed algorithm works efficiently and minimize the delay as compared to the pre-timed traffic signal.

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