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Computational Analysis of Developing Laminar Flow in a Pipe

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ABSTRACT

A robust computational study is proposed for calculating the velocity and pressure distribution of steady, incompressible and developing laminar flow in a pipe. The study of fluid flow in the entrance region is very useful for many practical applications. Vorticity stream-function approach was used for this computational study. Poisson and vorticity-transport equations were used to find the velocity distribution in pipe. Dimensional equations are converted into dimensionless form, in order to insureflexibility of this work on pipe dimension and fluid properties. Finite volume method and upwind scheme used to solve the dimensionless form of Poisson and vorticity-transport equation.Computer program (C language) was written to solve the dimensionless form of equations to calculate the velocity distribution and pressure distribution. In the developing region velocity profile and pressure drop were studied. The computed results were compared with Hagen-Poiseuille equation and to studied effect of Reynolds number on pressure drop.

KEYWORDS

Computational Study, Vorticity, Stream Function, Developing Flow, Pressure Drop, Velocity Distribution.

Nomenclature

a	Aspect ratio of pipe radius to length	D	Diameter of pipe (m)
L	Length of the pipe (m)	P	Pressure (N/m^2)
P^*	Dimensionless pressure	r	Radial coordinate
R	Radius of pipe (m)	Re	Reynolds number
U_m	Bulk Velocity (m/s)	u_r	Radial velocity (m/s)
u_r*	Dimensionless radial velocity	u_z	Axial Velocity (m/s)
u_z*	Dimensionless axial velocity	V	Velocity of fluid (m/s)
z	Axial coordinate	ρ	Density of fluid (kg/m^3)
μ	Dynamic viscosity of fluid (kg/m-s)	ν	Kinematic viscosity (m^2/s)
η	Dimensionless axial coordinate	ξ	Dimensionless radial coordinate
ψ	Stream function (m^3/s)	Ω	Vorticity (S^{-1})
ψ*	Dimensionless stream function	Ω*	Dimensionless vorticity
∇	Vector operator		
Subscripts			
e	East interface	n	North interface
s	South interface	w	West interface
E	East node	W	West node
N	North node	S	South node
P	Central node		
Superscripts			
*	Dimensionless quantity		

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Introduction

Fluid flow in circular and noncircular pipes in commonly encountered in practice. The hot and cold water that we use in our homes is pumped through pipes. Oil and natural gas are transported hundreds of mile by large pipelines. Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications and fluid distribution networks. The fluid in such applications is usually forced to flow by a fan or pump through a flow section. A typical piping system involves pipes of different diameters connected to each other by various fittings or elbows to route to the fluid, valves to control the flow rate, and pumps to pressurize the fluid. Most of fluids are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion.

One of the classical flow treated in introductory fluid mechanics lectures are laminar, fully developed pipe flow, in other words, the so-called Hagen-Poiseuille flow. The development of these flows from pre-assigned velocity profiles requires certain axial distances from the pipe. Irrespective of a particular inlet velocity profile or what happens in detail at the entrance of a pipe, the physical mechanisms behind such axial development of a flow are well established and understood. Owing to the no-slip velocity condition at walls, the fluid next to the wall is immediately slowed as soon as the flow enters a pipe. This retardation near the wall spreads inwards owing to viscous effects and the slowed-down fluid close to the wall causes the fluid in the centre to move faster, since the cross-sectional mass flow rate at any axial location remains constant. Ultimately, moving in the flow direction, the fully developed state of the flow is reached, the parabolic velocity distribution of the Hagen-Poiseuille flow develops. The closest location from the entrance where this phenomenon occurs defines the hydrodynamic entrance length as the distance from the inlet of the pipe to the location of the fully developed pipe flow.

The entrance length, pressure gradient and velocity profile developments along the pipe length have been interest and concern of many engineers and scientists over the years. The problem is somewhat difficult and sensitive of nature. To simplify the problem by some assumptions and the accuracy of the solution will no doubt depend on these assumptions as well as on the method adopted. Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe. The developing flow problems are very difficult to solve theoretically, so most of the developing flow problems are solve by numerically. The main objective of this project work is to numerically determine the development of dimensionless velocity and pressure distribution in the developing laminar flow of the pipe and to study the effect of Reynolds number in pressure drop. The study of fluid flow in the entrance region of a pipe is very useful for so many practical applications such as a short pipe leading to a diffuser or nozzle, the cooling water in an engine is transported by hoses to the pipes in the radiator where it is cooled as it flows, and the short pipe in fluid distribution networks. The present works deals with how the velocity and pressure varies with the developing laminar flow in a pipe.

Literature Survey

A fluid entering a circular pipe at a uniform velocity, Because of the no-slip condition, the fluid particles in the layer in contact with the surface of the pipe come to a complete stop. This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the pipe has to increase to keep the mass flow rate through the pipe constant. As a result, a velocity gradient develops along the pipeThe Region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the velocity boundary layer or just the boundary layer [2, 6]. The hypothetical boundary surface divides the flow in a pipe into two regions: the boundary layer region, in which the viscous effects and the velocity changes are significant, and the irrotational flow region, in which the frictional effects are negligible and the velocity remains essentially constant in the radial direction. The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the pipe centre and thus fills the entire pipe, as shown in Fig. 1.The region from the pipe inlet to the point at which the boundary layer merges at the centre line is called the hydrodynamic entrance region, and the length of this region is called the hydrodynamic entry length. Flow in the entrance region is called hydro-dynamically developing flow since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the hydro-dynamically fully developed region. The flow is said to be fully developed when the normalized temperature profile remains unchanged as well. Hydro-dynamically developed flow is equivalent to fully developed flow when the fluid in the pipe is not heated or cooled since the temperature in this case

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remains essentially constant throughout. The velocity profile in the fully developed region is parabolic in laminar flow.

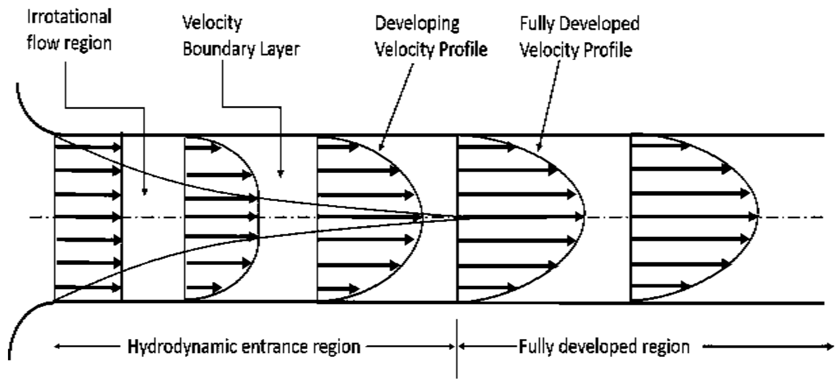

Fig. 1. Fully Developed flow in pipe

Maciej Matyka [1] reported solution to incompressible Navier - Stokes equations in non - dimensional form by Vorticity-Stream function approach are compared and results of them are analysed for standard CFD test case - drived cavity flow. Different aspect ratios of cavity and different Reynolds numbers are studied.

Salih.A [2] streamfunction-vorticity formulation was among the first unsteady, incompressible Naiver-Stokes algorithms. The original finite difference algorithm was developed by from [1] at Los Alamos laboratory. For incompressible two-dimensional flows with constant fluid properties, the Naiver-Stokes equations can be simplified by introducing the stream-function and vorticity was dependent variables.

Francois Dubois, Michel Salaun [3] studied numerically the Stokes problem of incompressible fluid dynamics in two and three-dimension spaces, for general bounded domains with smooth boundary. We use the vorticity–velocity-pressure formulation and introduce a new Hilbert space for the vorticity.

Sun Kyoung Kim [4] analysed the fully developed laminar flow of the Cross fluid between parallel plates under uniform heat flux. The formulation for the Nusselt number has been derived based on the analytically described velocity and flow rate. The velocity has been obtained analytically in terms of the shear rate.

Shah and London [5] studied analytically the laminar forced convection flow in straight and curved. Analyses were performed for curved ducts of circular, rectangular, elliptical and annular cross sections and valuable correlations for friction factor and Nusselt number were deduced.

Saffari et al. [6] studied experimentally and numerically the hydrodynamic entrance length of single and two-phase bubbly flow in helical coils. It was shown that the entrance length increases with the increase of the pipe diameter and decreases with the increase of the coil diameter. From the other side,

Prabhanjan [7] studied experimentally the effect of coil configuration on the heat transfer rate aiming at developing correlations that relate the coil parameters with heat transfer to the in terms of dimensionless numbers.

In general most of the time N-S equation are solved by numerically because non linearity involved in convective terms**,** pressure gradient acts as source term in momentum equation, but there is no separate equation to find out it**,** no pressure terms in continuity equation and simultaneously solving three partial differential equations is very difficult. There are many numerical methods available to solve the steady, unsteady, compressible and incompressible flow momentum equations that is artificialcompressibility method, pressure correction method and density based solver, vorticity-stream function approach [8]. In this work the problem is solved by vorticity and stream function approach and the main motive is to ensure the closeness of velocity distribution at the end of the pipe is almost same as the value obtained from Hagen-Poiseuille law.

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GoverningEquation

Laminar flow in pipe with circular cross section has been studied extensively as presented in the literature. The convenient coordinate system that best suit the present geometrical configuration is (r, θ, z). For steady, laminar, axisymmetric and incompressible flow of a Newtonian fluid inside a pipe, the governing equations are
Continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{\partial u_z}{\partial z} = 0 \ (1)$$

r-momentum equation:

$$u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u_r}{\partial z^2}\right) \ (2)$$

z-momentum equation:

$$u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right) \ (3)$$

Problem Formulation

The problem of interest is how the velocity and pressure change in a steady, laminar, incompressible and developing pipe flow shown in fig. 2. For the baseline case, at inlet of the pipe the bulk velocity U_m = 0.01 m/s. Pipe dimensions are: radius R = 0.05 m and length L = 0.5 m. The fluid kinematic viscosity $\nu = 1 \times 10^{-5}$ m^2/s.

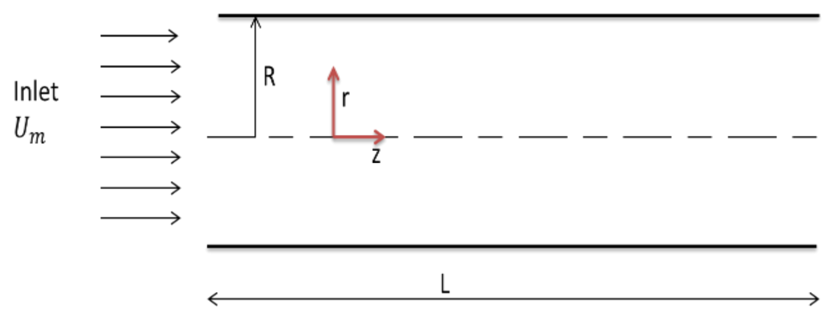

Fig. 2. Problem description

Vorticity-Streamfunction$(\boldsymbol{\Omega} - \boldsymbol{\psi})$ Approach

The Vorticity stream function formulation was among first unsteady, incompressible Naiver-Stokes algorithms. Naiver-Stokes equations can be simplified by introducing the stream-function ψ and vorticity $\vec{\Omega}$ as dependent variables. It is an effective and popular approach for two dimensional steady, incompressible Naiver-Stokes equations.
Velocity Stream function Relation.

$$u_r = \frac{-1}{r}\frac{\partial \psi}{\partial z} \ ; \ u_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \ (4)$$

Vorticity StreamfunctionRelation (Poisson Equation)

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{r}\frac{\partial \psi}{\partial z}\right) = -\Omega_\theta \ (5)$$

Vorticity Transport Equation

$$u_r\frac{\partial \Omega}{\partial r} + u_z\frac{\partial \Omega}{\partial z} = \frac{\Omega u_r}{r} + \nu\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(\mathrm{r}\Omega)\right] + \nu\frac{\partial^2 \Omega}{\partial z^2} \ (6)$$

The dimensional form of Poisson and vorticity transport equation depends on object dimensions and fluid properties which limit its application for obtaining general solution for any dimension and any material. To insure the independency of present work over pipe dimensions and fluid properties the dimensional Poisson and vorticity transport equation is converted into non-dimensional form by using dimensionless parameters. The non-dimensional terms as follows.

$$\xi = \frac{r}{R}, 0 \leq \xi \leq 1.0 \ (7) \eta = \frac{z}{L}, 0 \leq \eta \leq 1.0 \ (8)$$
$$u_r^* = \frac{u_r}{U_m} \ ; \ u_z^* = \frac{u_z}{U_m} \ ; \ \psi^* = \frac{2\psi}{U_m R^2} \ ;$$
$$\Omega^* = \frac{\Omega R^2}{\nu} \ ; \ P^* = \frac{P}{\rho U_m^2}$$

Where,

R is radius, L is length and U_m is inlet bulk velocity of the pipe, ψ is streamfunction, Ω is vorticity, u_r^* is dimensionless radial velocity,u_z^* is dimensionless axial velocity, $\psi^*$is dimensionless stream-function, Ω^* is dimensionless vorticity and ξ, η are the dimensionless *r* and *z* coordinates, respectively.

Dimensionless form of velocity streamfunctionrelation, Poisson and vorticity transport equations as bellows.

$$u_r^* = \frac{-1}{2} \frac{R}{L\xi} \frac{\partial \psi^*}{\partial \eta} \ (9)$$
$$u_z^* = \frac{1}{2\xi} \frac{\partial \psi^*}{\partial \xi} \ (10)$$
$$\frac{\partial}{\partial \xi}\left(\frac{1}{\xi}\frac{\partial \psi^*}{\partial \xi}\right) + \frac{a^2}{\xi}\frac{\partial}{\partial \eta}\left(\frac{\partial \psi^*}{\partial \eta}\right) = \frac{-4\Omega^*}{Re} \ (11)$$
$$\frac{1}{2}u_r^* \frac{\partial \Omega^*}{\partial \xi} + \frac{a}{2}u_z^* \frac{\partial \Omega^*}{\partial \eta} = \frac{1}{2}\frac{\Omega^*}{\xi}u_r^* + \frac{1}{Re}\frac{\partial}{\partial \xi}\left[\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi\Omega^*)\right] + \frac{a^2}{Re}\frac{\partial^2 \Omega^*}{\partial \eta^2} \ (12)$$

Boundary Condition for stream function.

Some initial values of streamfunction and vorticity needed for solve the Poisson equation and vorticity transport equation. Pipe inlet has uniform velocity U_m (plug flow). At inlet of the pipe the bulk velocity is equal to axial velocity and radial velocity is zero.

$$u_z = U_m, u_r = 0 \ (13)$$
$$\psi^*(\xi, \eta) = \xi^2 \ (14)$$

The values of dimensionless stream function (ψ^*) will be zero at the axis and 1.0 at the wall. In-between it will vary square of dimensionless radial coordinate (ξ^2).

Boundary Conditions for Vorticity

Vorticity value of inlet is zero (because inlet is plug flow). Radial velocity at axis is zero and the axial velocity change with respect to *r* at axis is zero, so the vorticity value at axis are zero. Vorticity creates at pipe wall and comes in to flow

$$\psi_{i+1} = \psi_i + \left(\frac{\partial \psi}{\partial r}\right)_i \Delta r + \left(\frac{\partial^2 \psi}{\partial r^2}\right)_i \frac{(\Delta r)^2}{2} \ (15)$$

Node$i + 1$ – wall node
Node$i$ – below the wall node
Taylor series for node *i* in terms of node *i+1*

$$\psi_i = \psi_{i+1} - \left(\frac{\partial \psi}{\partial r}\right)_{i+1} \Delta r + \left(\frac{\partial^2 \psi}{\partial r^2}\right)_{i+1} \frac{(\Delta r)^2}{2} \ (16)$$

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$\Omega_{i+1}^{*} = \frac{Re}{2\xi}\frac{(\psi_{i+1}^{*}-\psi_{i}^{*})}{(\Delta\xi)^2}$ (17)Dimensionless form of momentum equations

$$u_r^*\frac{\partial u_r^*}{\partial \xi} + a u_z^*\frac{\partial u_r^*}{\partial \eta} = -\frac{\partial P^*}{\partial \xi} + \frac{2}{Re}\left[\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial u_r^*}{\partial \xi}\right) - \frac{u_r^*}{\xi^2}\right] + \frac{2a^2}{Re}\frac{\partial^2 u_r^*}{\partial \eta^2} \quad (18)$$

$$u_r^*\frac{\partial u_z^*}{\partial \xi} + a u_z^*\frac{\partial u_z^*}{\partial \eta} = -\frac{\partial P^*}{\partial \eta} + \frac{2}{Re}\left[\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial u_z^*}{\partial \xi}\right)\right] + \frac{2a^2}{Re}\frac{\partial^2 u_z^*}{\partial \eta^2} \quad (19)$$

In these momentum equations has only two unknowns (velocity & pressure).Once find the dimensionless axial and radial velocities by solving the dimensionless Poisson and vorticity transport equation, using those velocities in above equations to find out the dimensionless pressure drop in pipe.

Numerical Formulation

Simple one dimensional flow problems (Fully developed flow) with simple boundary conditions can be easily solved by analytical method. But many developing flow problems encountered in practice involved complicated geometries with complex boundary condition are not easy to solve by analytical method. In such cases, sufficiently accurate approximate solutions can be obtained by computers using a numerical method.

Analytical methods are based on solving the governing differential equations together with the boundary conditions. On the other hand, numerical methods are based on replacing the differential equation by a set of *n* algebraic equations for the unknown velocities and pressures at *n* selected points in the medium, and the simultaneous solution of these equations results in the velocity and pressure values at those discrete point.

For formulation of algebraic equation the pipe has been discretized into numbers of small cells by using various gridlines along *r* and *z* axes. The coordinate gas been chosen from the point of origin the *r* and *z* axes are taken positive in increasing direction and varies from 0 to 1.0 (as non-dimensional). The discretized pipe has been shown in fig. 3.After the discretization, it has been found that cells are classified into full cell (internal node) and half cell (boundary node).

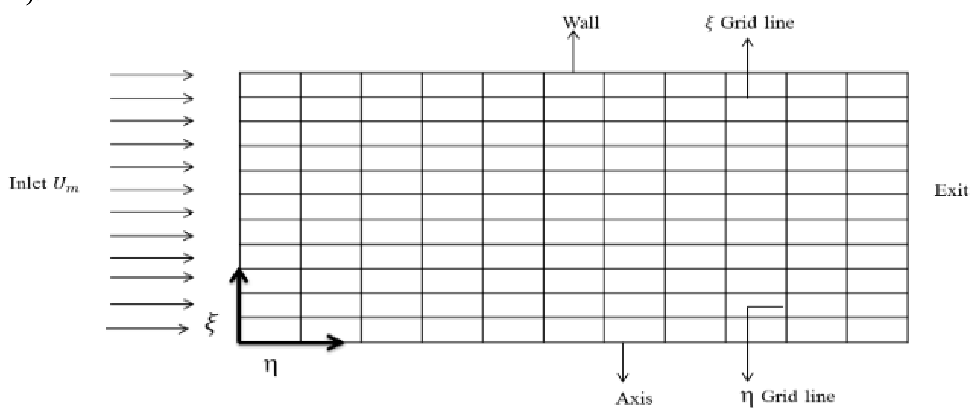

Fig. 1. Discretized Pipe

The algebraic equation has been formed for each cell by solving the dimensionless form of Poisson and vorticity transport equation numerically. Finite volume method has been used to solve the dimensionless form of Poisson and vorticity transport equation numerically.by referring the equations dimensionless form of Poisson and vorticity transport equations are

$$\frac{\partial}{\partial \xi}\left(\frac{1}{\xi}\frac{\partial \psi^*}{\partial \xi}\right) + \frac{a^2}{\xi}\frac{\partial}{\partial \eta}\left(\frac{\partial \psi^*}{\partial \eta}\right) = \frac{-4\Omega^*}{Re} \quad (20)$$

$$\frac{1}{2}u_r^*\frac{\partial \Omega^*}{\partial \xi} + \frac{a}{2}u_z^*\frac{\partial \Omega^*}{\partial \eta} = \frac{1}{2}\frac{\Omega^*}{\xi}u_r^* + \frac{1}{Re}\frac{\partial}{\partial \xi}\left[\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi\Omega^*)\right] + \frac{a^2}{Re}\frac{\partial^2 \Omega^*}{\partial \eta^2} \quad (21)$$

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Algebraic Formulation for Cells

After discretization it has been found that the some nodal points having the four neighbour nodes, as shown in fig. Central node P having four neighbour nodes namely east E and west W along z axis, north N and south S along r axis. This type of cell named as full cell as shown in fig. 4.

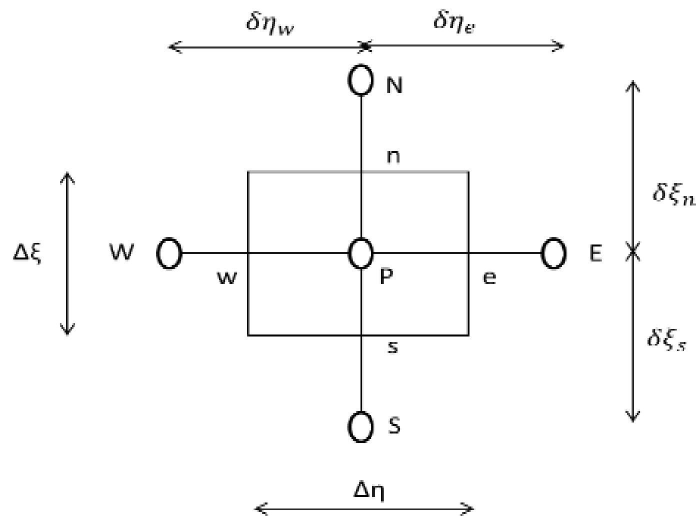

Fig. 2. Full Cell

Integrate across the typical control volume P for Poisson equation

$$\int_s^n \int_w^e \left[\frac{\partial}{\partial \xi}\left(\frac{1}{\xi}\frac{\partial \psi^*}{\partial \xi}\right) + \frac{R^2}{L^2}\frac{1}{\xi}\frac{\partial}{\partial \eta}\left(\frac{\partial \psi^*}{\partial \eta}\right)\right] d\eta\, d\xi = \int_s^n \int_w^e \left[\frac{-4\Omega^*}{Re}\right] d\eta\, d\xi \quad (22)$$

$$a_P\psi_P^* = a_E\psi_E^* + a_W\psi_W^* + a_N\psi_N^* + a_S\psi_S^* + b$$

Where

$$a_E = a^2 ln\left(\frac{\xi_n}{\xi_s}\right)\frac{1}{\delta\eta_e}\ ; a_W = a^2 ln\left(\frac{\xi_n}{\xi_s}\right)\frac{1}{\delta\eta_w}; a_N = \left(\frac{1}{\xi}\right)_n \frac{\Delta\eta}{\delta\xi_n}$$

$$a_S = \left(\frac{1}{\xi}\right)_s \frac{\Delta\eta}{\delta\xi_s}\ ;\ b = \frac{-4\Omega_P^*}{Re}\Delta\eta\Delta\xi$$

$$a_P = a_E + a_W + a_N + a_S$$

Before doing the integration of vorticitytransport equation combined with continuity equation.

Integrate across the typical control volume P

$$\int_s^n \int_w^e \left[\frac{\partial}{\partial \xi}(\xi u_r^*\Omega^*) + a\frac{\partial}{\partial \eta}(\xi u_z^*\Omega^*) = u_r^*\Omega^* + \frac{2\xi}{Re}\frac{\partial}{\partial \xi}\left[\frac{1}{\xi}\frac{\partial}{\partial \xi}(\xi\Omega^*)\right] + \frac{2a^2\xi}{Re}\frac{\partial^2\Omega^*}{\partial \eta^2}\right] d\eta\, d\xi \quad (23)$$

This is a convection diffusion type equation, using first order upwind scheme to solve and get the algebraic equation,

$$a_P\Omega_P^* = a_E\Omega_E^* + a_W\Omega_W^* + a_N\Omega_N^* + a_S\Omega_S^*$$

$$a_E = \frac{a^2}{Re}\left(\frac{\xi_n^2 - \xi_s^2}{\delta\eta_e}\right) + a\xi_P\llbracket -u_{z,e}^*, 0\rrbracket\Delta\xi$$

$$a_W = \frac{a^2}{Re}\left(\frac{\xi_n^2 - \xi_s^2}{\delta\eta_w}\right) + a\xi_P\llbracket u_{z,w}^*, 0\rrbracket\Delta\xi$$

$$a_N = \frac{2}{Re}\frac{\xi_n}{\delta\xi_n}\Delta\eta + \xi_n\llbracket -u_{r,n}^*, 0\rrbracket\Delta\eta$$

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$$a_S = \frac{2}{Re}\frac{\xi_s}{\delta\xi_s}\Delta\eta + \xi_s[\![u^*_{r,s}, 0]\!]\Delta\eta$$
$$a_P = \xi_n[\![u^*_{r,n}, 0]\!]\Delta\eta + \xi_s[\![-u^*_{r,s}, 0]\!]\Delta\eta + a\xi_P\left([\![u^*_{z,e}, 0]\!] + [\![-u^*_{z,w}, 0]\!]\right)\Delta\xi - u^*_{r,p}\,\Delta\eta\Delta\xi$$
$$+\frac{2}{Re}\left[\frac{\xi_n}{\delta\xi_n} + \frac{\xi_s}{\delta\xi_s} + ln\left(\frac{\xi_n}{\xi_s}\right)\right]\Delta\eta + \frac{a^2}{Re}(\xi_n^2 - \xi_s^2)\left[\frac{1}{\delta\eta_e} + \frac{1}{\delta\eta_w}\right]$$

After discretization it has been found that the some nodal points having the three neighbour nodes, these types of cells are named as half-cell. One of the typical half-cell has been shown in fig. 5. As shown in Figure of typical half-cell the Central node P having three neighbour nodes namely west W along z axis, north N and south S along r axis. The Poisson equation for half-cell changed into below algebraic equation,

$$a_P\psi^*_P = a_W\psi^*_W + a_N\psi^*_N + a_S\psi^*_S + b$$

Where

$$a_W = a^2 ln\left(\frac{\xi_n}{\xi_s}\right)\frac{1}{\delta\eta_w};\ a_N = \left(\frac{1}{\xi}\right)_n \frac{\Delta\eta}{2\delta\xi_n}\ ;\ a_S = \left(\frac{1}{\xi}\right)_s \frac{\Delta\eta}{2\delta\xi_s}$$
$$b = \frac{-4\Omega^*_P}{Re}\Delta\eta\Delta\xi\ ;\ a_P = a_W + a_N + a_S$$

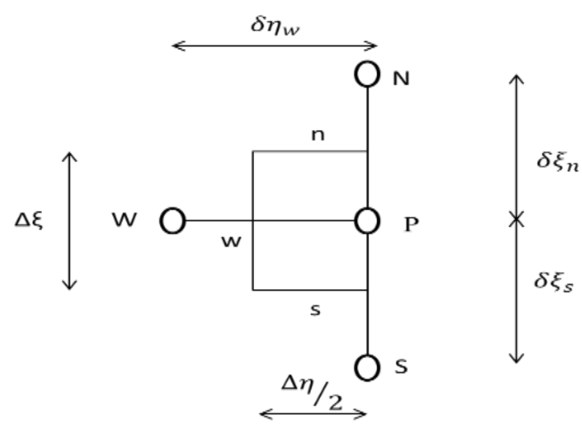

Fig. 5. Half Cell

Vorticity transport equation for half-cell changed into below algebraic equation,

$$a_P\Omega^*_P = a_W\Omega^*_W + a_N\Omega^*_N + a_S\Omega^*_S$$
$$a_W = \frac{a^2}{Re}\left(\frac{\xi_n^2 - \xi_s^2}{\delta\eta_w}\right) + a\xi_P[\![u^*_{z,w}, 0]\!]\Delta\xi$$
$$a_N = \frac{2}{Re}\frac{\xi_n}{\delta\xi_n}\frac{\Delta\eta}{2} + \xi_n[\![-u^*_{r,n}, 0]\!]\frac{\Delta\eta}{2}$$
$$a_S = \frac{2}{Re}\frac{\xi_s}{\delta\xi_s}\frac{\Delta\eta}{2} + \xi_s[\![u^*_{r,s}, 0]\!]\frac{\Delta\eta}{2}$$
$$a_P = \xi_n[\![u^*_{r,n}, 0]\!]\frac{\Delta\eta}{2} + \xi_s[\![-u^*_{r,s}, 0]\!]\frac{\Delta\eta}{2} + a\xi_P u^*_{z,P}\Delta\xi + a\xi_P\left([\![-u^*_{z,w}, 0]\!]\right)\Delta\xi - u^*_{r,p}\frac{\Delta\eta}{2}\Delta\xi$$
$$+\frac{2}{Re}\left[\frac{\xi_n}{\delta\xi_n} + \frac{\xi_s}{\delta\xi_s} + ln\left(\frac{\xi_n}{\xi_s}\right)\right]\frac{\Delta\eta}{2} + \frac{a^2}{Re}(\xi_n^2 - \xi_s^2)\left[\frac{1}{\delta\eta_w}\right]$$

Once find out the velocity distribution from solving the algebraic equation to substitute the corresponding values to momentum equation by using centraldifference scheme to solve and find out the pressure distribution along the pipe.

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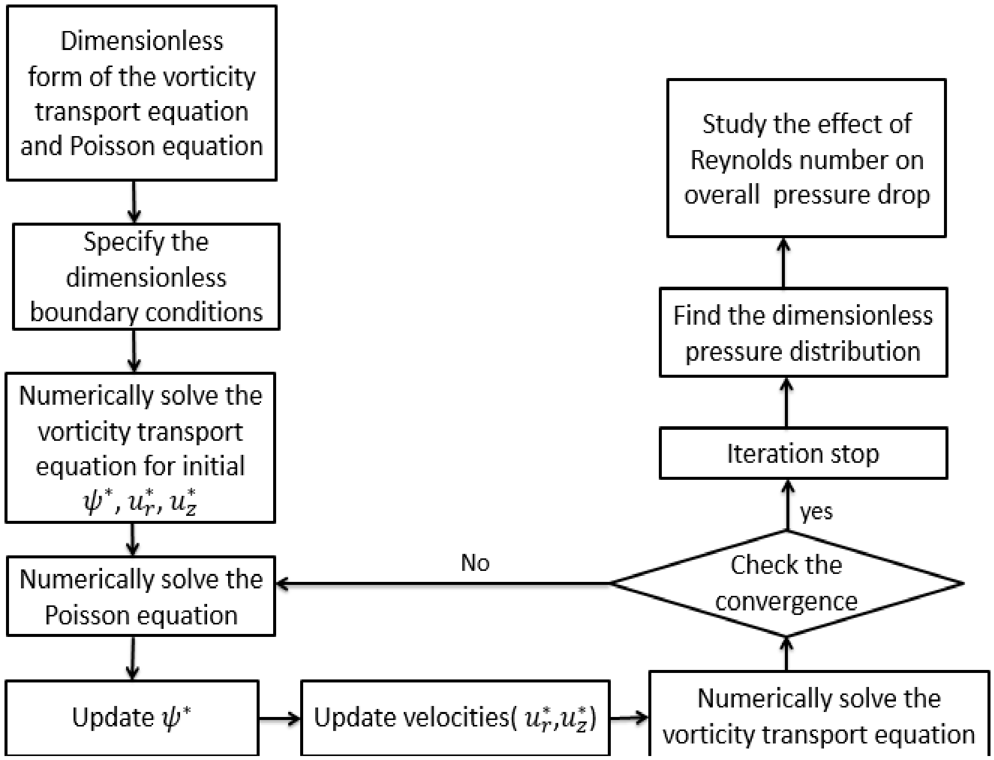
Flow Chart



Result and Discussion

In this study, how the velocity change in a steady, laminar, incompressible and developing pipe flow was investigated and then the results were compared with Hagen-Poiseuille law.

Table1. Dimensionless radial velocity at various nodal points

Radius (ξ)	**Length (η)=0.2**	**Length (η)=0.4**	**Length (η)=0.6**	**Length (η)=0.8**	**Length (η)=1.0**
1	0	0	0	0	0
0.9	0.0029	0.0007	0.0003	0.00016	0.00003
0.8	0.0097	0.0026	0.0011	0.00057	0.00008
0.7	0.0171	0.0051	0.0022	0.00114	0.00014
0.6	0.0222	0.0074	0.0034	0.00173	0.00018
0.5	0.0240	0.0090	0.0042	0.00219	0.00021
0.4	0.0225	0.0093	0.0046	0.00240	0.00021
0.3	0.0186	0.0083	0.0042	0.00226	0.00019
0.2	0.0132	0.0062	0.0032	0.00176	0.00014
0.1	0.0068	0.0033	0.0017	0.00097	0.00007
0	0	0	0	0	0

The computational result of radial velocity at some sequential points has been shown in Table.1 and graphical representation shown in fig. 6.Radial velocity at inlet, axis and wall of the pipe is zero. In developing region radial velocity continuously change by small amount, once the flow is fully developed (at exit of the pipe) the radial velocity is almost zero.

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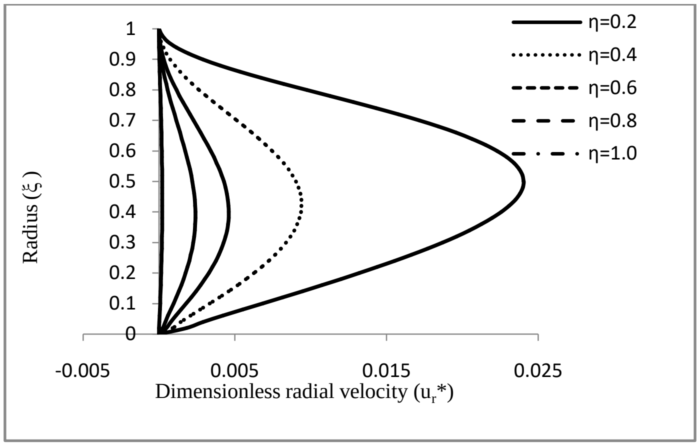

Fig. 6. Dimensionless Radial Velocity

Table 2. Dimensionless axial velocity at various nodal points

ξ	η=0.2	η=0.4	η=0.6	η=0.8	η=1.0
1	0	0	0	0	0
0.9	0.4669	0.4150	0.3974	0.3892	0.3853
0.8	0.8462	0.7734	0.7467	0.7340	0.7280
0.7	1.1285	1.0717	1.0463	1.0338	1.0279
0.6	1.3204	1.3079	1.2952	1.2880	1.2845
0.5	1.4408	1.4846	1.4937	1.4966	1.4978
0.4	1.5113	1.6091	1.6448	1.6611	1.6686
0.3	1.5502	1.6913	1.7531	1.7838	1.7984
0.2	1.5703	1.7416	1.8247	1.8679	1.8890
0.1	1.5797	1.7682	1.8650	1.9168	1.9424
0	1.5825	1.7767	1.8784	1.9333	1.9604

The computational result of axial velocity has been shown in Table. 2 and graphical representation shown in Fig 7. From that axial velocity profile continuously change in developing region; once the flow is fully developed (at the exit of the pipe) velocity profile remains unchanged. The velocity profile in the fully developed region is parabolic.

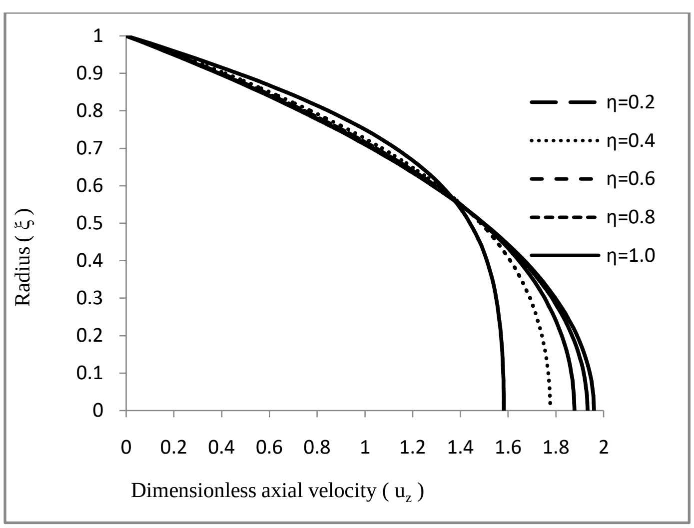

Fig. 7. Dimensionless Axial Velocity

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In a fully developed laminar flow each fluid particle moves at constant axial velocity along a stream line and the velocity profile doesn't changed in the flow direction. There is no motion in the radial direction and thus the velocity component in the direction normal to the floe everywhere is zero. For the baseline case the length of the pipe is calculated from fully developed length formulae and the end of the pipe flow is changed to fully develop. Using that corresponding length the axial velocity at the end of the pipe is compared with the velocities obtained from Hagen-Poiseuille law.

$$u_z(r) = 2U_m\left(1 - \frac{r^2}{R^2}\right)$$

Maximum velocity occurs at axis of the pipe,$2U_m = u_{max}$
Dimensionless form$u_z^* = 2(1 - \xi^2)$

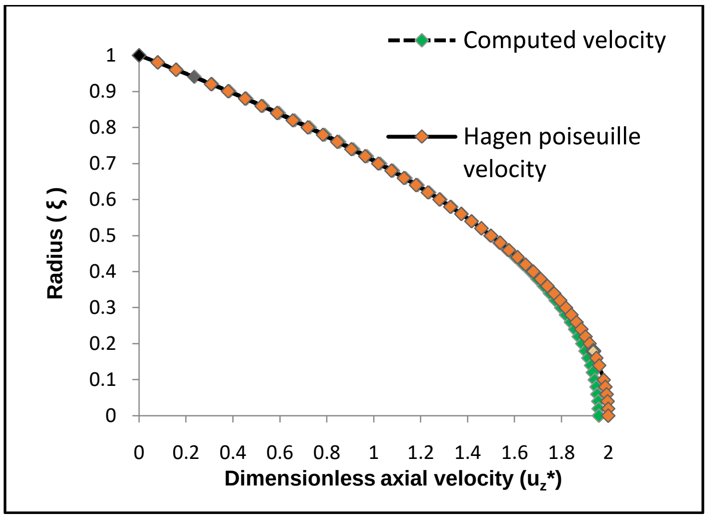

Fig.8. Comparison of axial velocity

In fully developed flow the maximum axial velocity is two times of bulk velocity. Non dimensional inlet axial velocity (bulk velocity) of the pipe is 1. Once the flow reaches developed region the radial velocity is almost zero. From the computational analysis the maximum value of dimensionless velocity is 1.9605 at the axis of the pipe. This value is almost close to two times of bulk velocity. The velocity profile obtained from computational analysis compared with the velocity profile obtained from Hagen-Poiseuille law has been shown in fig. 8.Both the profiles are very similar so the computational value almost same as the standard value.

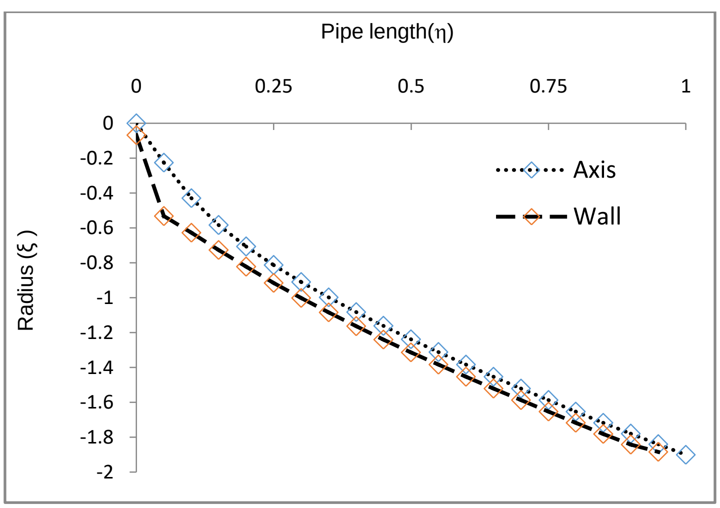

Fig.9. Pressure Drop

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Fluid flow in the hydrodynamic entrance region of a pipe the wall shear stress is the highest at the pipe inlet where the thickness of boundary layer is smallest, and decreases gradually to the fully developed value. Therefore the pressure drop is higher in entrance regions of a pipe. The dimensionless pressure distribution at pipe wall and axis along the length has been shown in fig. 9. From the figure the pressure gradient is more in developing region compared to flow towards the developed region. The dimensionless pressure values at end of pipe (developed region) from computational analysis is compared with Hagen-Poiseuille law for check the correctness of pressure values in developed region.

From figure $\frac{\Delta p}{\Delta z} = -0.3202\, N/m^3$

From Hagen-Poiseuille law $\frac{dp}{dz} = \frac{-8\mu U_m}{R^2} = -0.32\, N/m^3$

Both the pressure gradient values are almost same so the correctness of computational is good. The negative sign shows the drop in pressure because for the writing of code, initial pressure value of pipe inlet is zero.

Table 3. Pressure drop on various Re number

Bulk Velocity m/s)	**Reynolds number (Re)**	**Dimensionless Pressure drop (ΔP^*)**	**Dimensional Pressure drop (ΔP) (N/m^2)**
0.005	50	2.993699	0.07842
0.0075	75	2.72978	0.127845
0.01	100	1.8899864	0.189986
0.0125	125	1.664784	0.260123
0.015	150	1.498472	0.337156

The pipe dimensions and fluid properties are kept constant, only varying the Reynolds number by varying the bulk velocity. From computational analysis the axial and radial velocities are calculated by various Reynolds number and using that to find the dimensionless pressure drop. The dimensional pressure drop values are increasing by increase the Reynolds number shown in fig. 10.

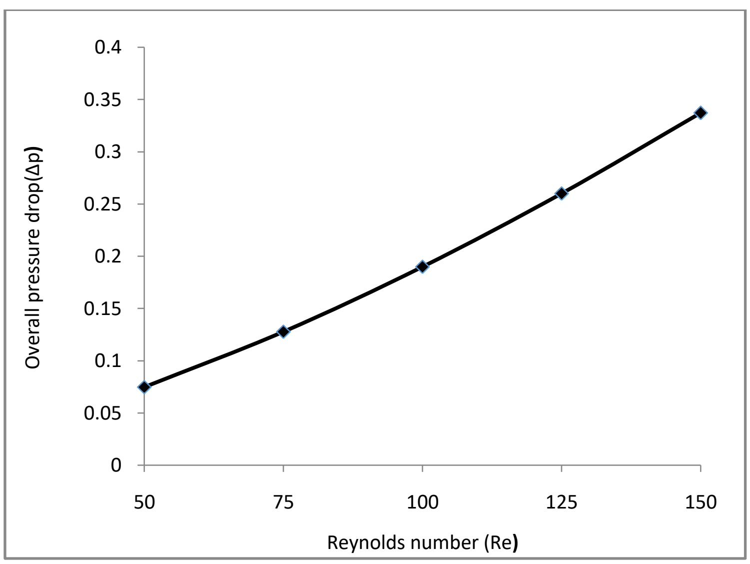

Fig.10. Pressure drop for various Re

Conclusions

The computational study has been carried out to determine the development of dimensionless velocity and pressure distribution in a steady, incompressible, laminar developing flow in a pipe. The effect of Reynolds number on pressure drop has been studied by varying the inlet bulk velocity. This numerical study is very useful to find out the velocity and pressure values in any particular location of the pipe. During the numerical study of developing laminar flow in a pipe, the following conclusion were made.

- In developing region dimensionless radial velocity continuously changed and once flow reached the developed region the radial velocity was zero
- In developing region dimensionless axial velocity profile continuously changed and once the flow reached the developed region profile remains unchanged and shape like a parabola
- Dimensionless axial velocity profile in developed region from computational study was almost same as the velocity profile obtained from Hagen-Poiseuille law
- The dimensionless pressure gradient was more in developing region compared to the flow towards the developed region (end of pipe) andthe values from computational study was almost same as the values obtained Hagen-Poiseuille law
- The dimensional pressure drop is increased by increasing the values of Reynolds number.

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