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Stability Analysis of Complex- Valued Memristive Neural Networks with Delay-Dependent Discrete-Time Cases

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ABSTRACT

Theoretically, the memristor as the fourth basic circuit element, was firstly postulated by Chua based on the integral theory of fundamental circuit in 1971. It has the unique electrical characteristics relative to resistor, capacitor and inductor. In 2008, researchers at HP's Laboratory implemented the physical model of the memristor, which means that it opens up new horizons for further development on circuit design. In 2009, the adaptive behavior of cells, which was similar to the property of the memristor, was proposed by means of the single-celled amoeba experiment. Based on the experimental verification, more research results show that artificial neural networks with variable weights constructed by the memristor can better simulate human brain like associative memory functions.

However, as an extension of RNNs, the main challenges we face are how to address the problems of complex-valued states and connection weights, especially complex-valued activation functions. Based on the Liouville's theorem, the activation function in CNNs cannot be both bounded and analytic while it's usually chosen to be a smooth bounded function in RNNs. The other way doesn't need to divide into two parts but should satisfy the Lipschitz continuity. For example, some complex-valued activation functions can't be divided into two parts, and some are discontinuous. As is known to us, when the system is discontinuous, it's difficult to ensure the stability of system. On the other hand, it's clear that the delay-dependent stability of neural networks(NNs) are less conservative than delay-independent ones, since time-delay phenomena are often encountered in various practical situations and may have negative effect on system stability.

Keywords: Memristor-Neural network – Lyapunov-Krasovskii – Discrete time – Complex valued function-equilibrium

INTRODUCTION

Mean while, a series of results have been acquired on the delay-dependent stability of Discrete-time NNs the Lyapunov-Krasovskii functional and linear matrix inequalities (LMIs). In this paper, an extended matrix inequality is proved to guarantee the delay-dependent stability of discrete-time MCVNNs. Hence, the dynamical behaviors of discrete-time MCVNNs are also analyzed in this article.

Model description and preliminaries

In this section, a class of memristor-based neural networks (MNNs) are introduced systematically. By Kirchhoff's current law, the ith subsystem of MNNs can be written as

$$\dot{v}_i(t) = -d_i v_i(t) + \sum_{j=1}^{n} a_{ij}\left(v_i(t)\right) f_j\left(v_j(t)\right) + \sum_{j=1}^{n} b_{ij}\left(v_i(t)\right) f_j\left(v_j(t-T)\right) + u_i, t \geq 0, \quad (1)$$

Where $i \in \mathfrak{L} = \{1,2,\ldots..n\}$, n corresponds to the number of units in the neural network; $v_i(t)$ is the voltage of the capacitor $\mathfrak{C}_i$; $d_i > 0$ represents the neuron self-inhibitions;$f_j\left(v_j(t)\right), f_j\left(v_j(t-T)\right)$ are the functions without and with time delays; $T(t)$ corresponds to the time delay and $0 \leq T_1 \leq T(t)T_2$; u_i denotes the external input or bais, $a_{ij}(.), b_{ij}(.)$ are the memristor-based weights given by

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$$a_{ij}(v_i(t)) = \frac{\mathfrak{M}_{ij}}{\mathfrak{C}_i} \times sgn_{ij}, b_{ij}(v_i(t)) = \frac{\widetilde{\mathfrak{M}}_{ij}}{\mathfrak{C}_i} \times sgn_{ij}$$

$$sgn_{ij} = \begin{cases} 1, & i \neq j \\ -1, & i = j \end{cases}$$

Where $\mathfrak{M}_{ij}$ is the memristor between the feedback function $f_j\left(v_j(t)\right)$ and $v_i(t)$; $\widetilde{\mathfrak{M}}_{ij}$ is the memristor between the feedback function $f_j\left(v_j(t-T)\right)$ and $v_i(t)$.

In this paper, we will consider complex-valued networks due to its extensive applications. In the complex domain, complex-valued states, connection weights, and activation functions exist in the MNNs, and MCVNNs with time delays can be written as follows from (1)

$$\dot{z}_i(t) = -d_i z_i(t) + \sum_{j=1}^{n} a_{ij}\left(z_i(t)\right) f_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij}\left(z_i(t)\right) f_j\left(z_j(t-T)\right) + u_i \qquad (2)$$

The above system (2) also can be rewritten as matrix from

$$\dot{z}(t) = -Ez(t) + A\big(z(t)\big)f\big(z(t)\big) + B\big(z(t)\big)f\Big(z\big(t-T(t)\big)\Big) + u$$
$$= F\Big(t, z(t), z\big(t-T(t)\big)\Big) \qquad (3)$$

Where

$$z(t) = \left(z_1(t), z_2(t), \ldots, z_n(t)\right)^T \in \mathbb{C}^n; D = diag(d_1, d_2, \ldots, d_n)^T \in \mathbb{R}^{n\times n}$$
$$A\big(z(t)\big) = \Big(a_{ij}\big(z_i(t)\big)\Big) \in \mathbb{C}^{n\times n}, B\big(z(t)\big) = \Big(b_{ij}\big(z_i(t)\big)\Big) \in \mathbb{C}^{n\times n}$$
$$f(.) = (f_1(.), f_2(.), \ldots, f_n(.))^T : \mathbb{C}^n \to \mathbb{C}^n \text{ and } u = (u_1, u_2, \ldots, u_n)^T \in \mathbb{C}^n$$

the feature of memristor and current voltage characteristic complex-valued connection weights are generally defined as follows

$$a_{ij}\big(z_i(t)\big) = \begin{cases} \hat{a}_{ij}, |z_i(t)| < \pi_j \\ \check{a}_{ij}, |z_i(t)| > \pi_i \end{cases}$$
$$b_{ij}\big(z_i(t)\big) = \begin{cases} \hat{b}_{ij}, |z_i(t)| < \pi_i \\ \check{b}_{ij}, |z_i(t)| > \pi_i \end{cases}$$

Where the switching jumps $\pi_i > 0, \hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}$ and $\check{b}_{ij}$ are constant numbers.The initial condition combined with system (2) is given by

$$z_i(\vartheta) = \psi_i(\vartheta), \vartheta \in [-T_2, 0], i \in \mathfrak{L}$$

BASIC DEFINITIONS

Definition 1

The linear mapping from the complex space to the real space is defined as follows

For $z \in \mathbb{C}^n$, let

$$\varphi(z) = \begin{pmatrix} Re(z) \\ Im(z) \end{pmatrix} \in \mathbb{R}^{2n}$$

For $A \in \mathbb{C}^{m\times n} (n > 1)$, let

$$\varphi(A) = \begin{pmatrix} Re(A) & -Im(A) \\ Im(A) & Re(A) \end{pmatrix} \in \mathbb{R}^{2m\times 2n}$$

Definition 2

For the system $\dot{x}(t) = g(t, x_t)$ with discontinuous right-hand sides, a set-valued map $\mathcal{G}(t, x_t): \mathbb{R} \times C \to 2^{\mathbb{R}^n}$ defined as

$$\mathcal{G}(t, x_t) \triangleq \bigcap_{\delta>0} \bigcap_{\mu(\mathcal{N})=0} \overline{co}[g(\mathcal{B}(x_t, \delta) \backslash \mathcal{N}]$$

Where $i = 1,2$ and $2^{\mathbb{R}^n}$ denotes the set of subsets of $\mathbb{R}^n$. $\overline{co}[[X]$ is the closure of the convex hull of X. $\mu(\mathcal{N})$ is the Lebesgue measure of set $\mathcal{N}$. $\mathcal{B}(x_t, \delta)$ represents the open ball with radius δ centered at x_t. A solution of the system in the sense of Filippov, with the initial condition $x(\vartheta) = \psi(\vartheta), \vartheta \in [-T_2, 0]$ is an absolutely continuous vector value

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function x(t) on any compact subinterval of $[0,t_1], t_1 \in (0,+\infty]$, which satisfies the differential inclusion $\dot{x}(t) \in G(t,x_t)$,a.e t $\in[0,t_1)$.
We given the Filippov solutions of system

$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi\left(A(z(t))\right)\varphi\left(f(z(t))\right) + \varphi\left(B(z(t))\right)\varphi\left(f\left(z(t-T(t))\right)\right) + \varphi(u)$$

For any compact subinterval of $[0,t_1), t_1 \in (0,+\infty]$, the Filippov solutions $\varphi(z(t))$,with the initial condition $\varphi(z(\vartheta)) = \varphi(\varphi_i(\vartheta)), \vartheta \in [-T_2, 0]$ satisfies the following differential inclusion

$$\varphi(\dot{z}(t)) \in \mathcal{F}(t,z_t), \text{ a.e } t \in[0,t_1) \quad (4)$$

Where,

$$\mathcal{F}(t,z_t) = (\mathcal{F}^T(t,z_t^R), (\mathcal{F}^T(t,z_t^I))^T$$

$$\mathcal{F}(t,z_t^R) = \bigcap_{\delta>0}\bigcap_{\mu(\mathcal{N})=0} \overline{co}[F^R(\mathcal{B}(z_t^R,\delta)\backslash\mathcal{N}]$$

and

$$\mathcal{F}(t,z_t^R) = \bigcap_{\delta>0}\bigcap_{\mu(\mathcal{N})=0} \overline{co}[F^R(\mathcal{B}(z_t^R,\delta)\backslash\mathcal{N}]$$

From the above analysis, we shall investigate dynamical behaviors of the differential inclusion (4) instead of system (2).

Definition 3

A set-valued map F with nonempty values is said to be upper semi continuous at $x_0 \in X$ if, for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$.

In view of whether the activation functions is continuous, we given the following two assumptions
$(H_1)'$ the activation function $f(.)$ satisfies

$$l_i'' \le \frac{\varphi_i(f(z)) - \varphi_i(f(w))}{\varphi_i(z) - \varphi_i(w)} \le l_i', \forall z, w \in \mathbb{C}^n, z \ne w$$

Where i-1,2,…,2n, and l_i', l_i'' are some constants. $(H_1)$the activation functions $f_i(.) = f_i^R(.) + f_i^I, f_i^R(.), f_i^I(.)$ are piecewise continuous.

i.e., $f_i^R(.), f_i^I(.)$ are continuous in $\mathbb{R}$ except a countable set of points $\rho_k$and $\bar{\rho}_k$of discontinuity, where there exists finite right and left limits, respectively; Moreover,$f_i^R(.), f_i^I(.)$ have a finite number of discontinuities on any compact interval of $\mathbb{R}$.

On the one hand, if activation functions in system (4) Satisfy the assumption $(H_1)'$, from set-valued maps theories and differential inclusions, system (4) can be written as the following differential inclusion

$$\varphi(\dot{z}(t)) \in -\varphi(E)\varphi(z(t)) + \overline{co}\left[\varphi\left(A(z(t))\right)\right]\varphi\left(f(z(t))\right) + \overline{co}\left[\varphi\left(B(z(t))\right)\right]\varphi\left(f(z(t-T(t)))\right) + \varphi(u)$$

Moreover, there exists measurable function matrices $A^R \in \overline{co}\{\breve{A}^R, \widehat{A}^R\}, A^I \in \overline{co}\{\breve{A}^I, \widehat{A}^I\}, B^R \in \overline{co}\{\breve{B}^R, \widehat{B}^R\}, A^I \in \overline{co}\{\breve{B}^I, \widehat{B}^I\}$, such that

$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi\left(f(z(t))\right) + \varphi(B)\varphi\left(f(z(t-T(t)))\right) + \varphi(u) \quad (5)$$

Where

$$\varphi(A) = \begin{pmatrix} A^R & -A^I \\ A^I & A^R \end{pmatrix}, \varphi(B) = \begin{pmatrix} B^R & -B^I \\ B^I & B^R \end{pmatrix},$$

and

$$\breve{A}^R = (\breve{a}_{ij}^R)_{n\times n}, \widehat{A}^R = (\hat{a}_{ij}^R)_{n\times n}, \breve{A}^I = (\breve{a}_{ij}^I)_{n\times n}, \widehat{A}^I = (\hat{a}_{ij}^I)_{n\times n}, B^R = (\breve{b}_{ij}^R)_{n\times n}, \widehat{B}^R = (b_{ij}^R)_{n\times n}, \breve{B}^I = (\breve{b}_{ij}^I)_{n\times n}, \widehat{B}^I = (\hat{b}_{ij}^I)_{n\times n}$$

On the other hand, if activation functions in system (4) satisfy the assumption (H_1), from set-valued maps theories and differential inclusion, system (4) can be written as the following differential inclusion

$$\varphi(\dot{z}(t)) \in -\varphi(E)\varphi(z(t)) + \overline{co}\left[\varphi\left(A(z(t))\right)\right]\varphi\left(f(z(t))\right) + \overline{co}\left[\varphi\left(B(z(t))\right)\right]\varphi\left(f\left(z(t-T(t))\right)\right) + \varphi(u)$$

Further, system (3.4) also can be rewritten as

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$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(\gamma(t)) + \varphi(B)\varphi\left(\gamma(t - T(t))\right) + \varphi(u) \qquad (6)$$

Where

$$\varphi(\gamma(t)) = \begin{pmatrix} \varsigma^R(t) \\ \eta^I(t) \end{pmatrix}, \varphi\left(\gamma(t - T(t))\right) = \begin{pmatrix} \varsigma^R(t - T(t)) \\ \eta^I(t - T(t)) \end{pmatrix}$$

And

$$\varsigma^R(.) \in \overline{co}[f^R(z(.))], \eta^I(.) \in \overline{co}[f^I(z(.))].$$

Definition 7

$\varphi(\overline{z})$ is an equilibrium point of system

$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(\gamma(t)) + \varphi(B)\varphi\left(\gamma(t - T(t))\right) + \varphi(u)$$

if and only if

$$\dot{\varphi}(z(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(\alpha(t)) + \varphi(B)\varphi(\alpha(t - T(t))) + \varphi(u)$$

$$0 \in -\varphi(E)\varphi(\overline{z}) + \overline{co}[\varphi(A(\overline{z})\varphi(f(\overline{z})))] + \overline{co}[\varphi(B(\overline{z})\varphi(f(\overline{z})))] + \varphi(u)$$

If $\varphi(\overline{z})$ is an equilibrium point of system, it turns out that there exists a vector $\varphi(\overline{\alpha}) \in \mathbb{R}^{2n}$ such that
$0 = -\varphi(E)\varphi(\overline{z}) + \varphi(A)\varphi(\overline{\alpha}) + \varphi(B)\varphi(\overline{\alpha}) + \varphi(u), \varphi(\overline{\alpha}) \in \overline{co}[\varphi(f(\overline{z}))]$
That is to say, $\varphi(\overline{\alpha})$ is an output equilibrium point of system

$$\dot{\varphi}(z(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(\alpha(t)) + \varphi(B)\varphi(\alpha(t - T(t))) + \varphi(u)$$

corresponding to $\varphi(\overline{z})$.

DISCRETE-TIME MCVNNs

While the continuous-time systems can't be an advantage for practical applications including digital computer and quadratic optimization, in this section, major objectives are to make the qualitative analysis on the discrete-time MCVNNs which are the counterparts of the continuous-time ones

$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(f\left(z\left(\left[\frac{t}{h}\right]h\right)\right)) + \varphi(B)\varphi(f\left(z\left(\left[\frac{t}{h}\right]h - \left[\tau\left(\left[\frac{t}{h}\right]h\right)\right]\right)\right) + \varphi(u), \qquad (7)$$

$$\varphi(\dot{z}(t)) = -\varphi(E)\varphi(z(t)) + \varphi(A)\varphi(\gamma(\left[\frac{t}{h}\right]h)) + \varphi(B)\varphi(\gamma(\left[\frac{t}{h}\right]h - [\tau(\left[\frac{t}{h}\right]h)])) + \varphi(u) \qquad (8)$$

for $t \in [nh, (n+1)h), n \in \mathbb{Z}^+$, where h is a fixed positive real number denoting a uniform discretization step-size. Clearly, for $t \in [nh, (n+1)h), n \in \mathbb{Z}^+$, we have $\left[\frac{t}{h}\right] = n$. Let $\left[T\left(\left[\frac{t}{h}\right]h\right)\right] = l(n), l_1 \leq l(n) \leq l_2, l_i \in \mathbb{Z}^+, i = 1,2$. For convenience in the following, we use the notation z (nh) = z(n). then, we can integrate (7) and (8) over [nh,t) separately and by allowing t→(n+1)h in the above, we obtain after some simplification that

$$\varphi(z(n+1)) = (\varphi(F) - \varphi(\theta(h)\,\varphi(E))\,\varphi(z(n)) + \varphi(\theta(h))\,\varphi(A)\,\varphi(f(z(n))) + \varphi(\theta(h))\,\varphi(B)\,\varphi(f(z(n-l(n)))) + \varphi(\theta(h))(u), \qquad (9)$$

$$\varphi(z(n+1)) = (\varphi(F) - \varphi(\theta(h))\,\varphi(E))\,\varphi(z(n)) + \varphi(\theta(h))\,\varphi(A)\,\varphi(\gamma(n)) + \varphi(\theta(h))\,\varphi(B)\,\varphi(f(\gamma(n-l(n))) + \varphi((\theta h))\,\varphi(u), \qquad (10)$$

For $n \in \mathbb{Z}^+$, where

$$\varphi(E) = \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix}, \qquad \varphi(\theta(h)) = \begin{pmatrix} \theta(h) & 0 \\ 0 & \theta(h) \end{pmatrix}$$

and $(h) = diag(\theta_1(h), \ldots \theta_i(h)), \theta_i(h) = \frac{1-e^{-d_i h}}{d_i}$,$i \in \mathfrak{J}$ At the same, the connection weights of the system can also be defined:

$$a_{ij}^R(z_i(n)) = \begin{cases} a_{ij}^{R'}, |z_i^R(n)| < \pi_i, \\ a_{ij}^{R''}, |z_i^R(n)| > \pi_i, \end{cases} \qquad a_{ij}^I(z_i(n)) = \begin{cases} a_{ij}^{I'}, |z_i^I(n)| < \pi_i, \\ a_{ij}^{I''}, |z_i^I(n)| > \pi_i, \end{cases}$$

$$b_{ij}^R(z_i(n)) = \begin{cases} b_{ij}^{R'}, |z_i^R(n)| < \pi_i, \\ b_{ij}^{R'\prime}, |z_i^R(n)| > \pi_i, \end{cases} \qquad b_{ij}^I(z_i(n)) = \begin{cases} b_{ij}^{I'}, |z_i^I(n)| < \pi_i, \\ b_{ij}^{I''}, |z_i^I(n)| > \pi_i. \end{cases}$$

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It is not difficult to verify that $\theta_i(h) > 0$ if $d_i > 0, h > 0$. the initial condition $z_i(\vartheta) = \psi_i(\vartheta), \vartheta \in [-l_2, 0]z$, system (9) and (10) are the discrete time analogues of the continuous time system (7) and (8),respectively.

Prior to carrying out the qualitative analysis of the discrete-time MCNNs, we first should give the existence interval and continuation of the solution, which is the prerequisite to study the stability of solutions.

Theorem 1

Suppose $(H)'$ is satisfied and there exists positive definite symmetric matrices $P_2 \in R^{6n\times 6n}$, $Q_i \in R^{4n\times 4n}$, $R_i \in R^{2n\times 2n}$, $i = 3,4$, positive-definite diagonal matrices $\check{J}_i \in R^{2n*2n}$, $i = 1,2,3,4$, and $\breve{K}_j \in R^{2n*2n}$, $j = 1,2,3$, such that

$$\bar{\Phi}(l(n)) = \bar{P}(l(n)) + \bar{Q} + \bar{R}(l(n)) + \bar{\mathcal{J}} + \bar{\mathcal{K}} < 0$$

Where

$$\bar{P}(l(n)) = -(\bar{\Gamma}_2 + l(n)\bar{\Gamma}_1)^T P_2(l(n)\bar{\Gamma}_1 + \bar{\Gamma}_2) + (\bar{\Gamma}_3 + l(n)\bar{\Gamma}_1)^T P_2(l(n)\bar{\Gamma}_1 + \bar{\Gamma}_3)$$

$$\bar{Q} = \operatorname{diag}(Q_3, Q_4 - Q_3, 0 - Q_4, Q_3, Q_4, Q_3, 0 - Q_4, 0,0,0)$$

$$R(l(n)) = e^{-T}(l_1^2 R_3, + l_{12}^2 R_2,)\bar{e} - \mathbb{E}_1 \tilde{R}_3 \mathbb{E}_1 - \begin{bmatrix} \mathbb{E}_3 \\ \mathbb{E}_2 \end{bmatrix}^T \begin{bmatrix} \tilde{R}_4 + \frac{l(n) - l_1}{l_{12}} \bar{T}_1 & S \\ * & \bar{R}_4 + \frac{l_2 - l(n)}{l_{12}} \bar{T}_2 \end{bmatrix} \begin{bmatrix} \mathbb{E}_3 \\ \mathbb{E}_2 \end{bmatrix}$$

$$\bar{\mathcal{J}} = \sum_{i=1}^{4} e_i^T \bar{L}\check{J}_i \bar{L} e_i - e_{i+4}^T \check{J}_i e_{i+4}$$

$$\bar{\mathcal{K}} = \sum_{i=1}^{3} (e_i^T - e_{i+1}^T)\bar{L}\breve{K}_i \bar{L}(e_i - e_{i+1}) - (e_{i+4}^T - e_{i+5}^T)\breve{K}_i(e_{i+4} - e_{i+5})$$

$$\bar{\Gamma}_1 = \begin{bmatrix} e_0 \\ e_0 \\ e_{10} - e_{11} \end{bmatrix}, \bar{\Gamma}_2 = \begin{bmatrix} e_1 \\ (l_1 + 1)e_9 - e_1 \\ (1 - l_1)e_{10} - e_{e3} + (l_2 + 1)e_{11} - e_2 \end{bmatrix}$$

$$\bar{\Gamma}_3 = \begin{bmatrix} (\varphi(E) - \varphi(\theta(h))\varphi(D))e_1 + \varphi(\theta(h))\varphi(A)e_5 + \varphi(\theta(h))\varphi(B)e_7 \\ (l_1 + 1)e_9 - e_2 \\ (1 - l_1)e_{10} - e_3 + (l_2 + 1)e_{11} - e_4 \end{bmatrix}$$

$$\mathbb{E}_i = \begin{bmatrix} e_i - e_{i+1} \\ e_i + e_{i+1} - 2e_{i+8} \end{bmatrix}, i = 1,2,3,$$

$$e_i = [0_{2n\times 2(i-1)n}, E_{2n\times 2n}, 0_{2n\times 2(11-i)n}]\, i = 1,2, \dots\dots\dots 11$$

$$\bar{e} = -\varphi(\theta(h))\varphi(D))e_1 + \varphi(\theta(h))\varphi(A)e_5 + \varphi(\theta(h))\varphi(B)e_7$$

$\check{J}_i = \operatorname{diag}(\check{J}_{1i}, \dots, \check{J}_{2ni})$, $i = 1,2,3,4$, $\quad \breve{K}_j = \operatorname{diag}(\breve{K}_{1i}, \dots, \breve{K}_{2ni})$, $i = 1,2,3$

Then, the system (9) is asymptotically stable for all time-varying delay.

Proof

To see this let us set

$$\tilde{z}(n) = z(n) - z*$$

Where z* is an equilibrium point of the system (28) if and only if

$$-\varphi(\theta(h))\varphi(D)\varphi(z^*) + \varphi(\theta(h))(\varphi(A) + \varphi(B))\varphi(f(z^*)) + \varphi(\theta(h))\varphi(u) = 0$$

It follows the above that the system (28) can be transformed to

$$\varphi(\tilde{z}(n+1)) = (\varphi(E) - \varphi(\theta(h))\varphi(D))\varphi(\tilde{z}(n)) + \varphi(\theta(h))\varphi(A)\varphi(\tilde{f}(\tilde{z}(n))) + \varphi(\theta(h))\varphi(B)\varphi(\tilde{f}(\tilde{z}(n - l(n)))$$

where $\varphi(\tilde{f}(\tilde{z}(.))) = \varphi(f(\tilde{z}(.) + z^*)) - \varphi(f(z^*))$

consider the ollowing lYAPUNOV–KRASOVSKII FUNCTIONAL

$$V(n) = V_1(n) + V_2(n) + V_3(n)$$

Where

$$V_1(n) = \xi_3^T(n) P_2 \xi_3(n)$$

$$V_2(n) = \sum_{i=n-l_1}^{n-1} \xi_4^T(i) Q_3 \xi_4(i) + \sum_{i=n-l_2}^{n-l_1-1} \xi_4^T(i) Q_4 \xi_4(i)$$

$$V_3(n) = l1 \sum_{j=-l_1}^{-1} \sum_{i=n+j}^{n-1} \Delta\varphi^T(\tilde{z}(i)) R_3 \Delta\varphi(\tilde{z}(i)) + l_{12} \sum_{j=-l_2}^{-l_1-1} \sum_{i=n+j}^{n-1} \Delta\varphi^T(\tilde{z}(i)) R_4 \Delta\varphi(\tilde{z}(i))$$

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With $P_2 > 0, Q_i > 0, R_i > 0, i = 3,4$ and

$$\xi_3(n) = \begin{bmatrix} \varphi(\tilde{z}(n)) \\ \sum_{i=n-l_1}^{n-1} \varphi(\tilde{z}(i)) \\ \sum_{i=n-l_2}^{n-l_1-1} \varphi(\tilde{z}(i)) \end{bmatrix}, \xi_4(i) = \begin{bmatrix} \varphi(\tilde{z}(i)) \\ \varphi((\tilde{z}(i))) \end{bmatrix}$$

Clearly, this functional is positive definite. Next we mainly prove $\Delta V(n) = V(n+1) - V(n)$ is negative defined combined with the vector

$$\bar{\zeta}(t) = [\bar{\zeta}_1^T(n), \bar{\zeta}_2^T(n), \bar{\eta}_1^T(n), \bar{\eta}_1^T(n), \bar{\zeta}_3^T(n)]^T$$

Where

$$\bar{\zeta}_1(n) = \begin{bmatrix} \varphi(\tilde{z}(n)) \\ \varphi(\tilde{z}(n-l_1)) \\ \varphi(\tilde{z}(n-l_2))) \end{bmatrix}, \bar{\zeta}_2(n) = \begin{bmatrix} \varphi\left(\tilde{f}(\tilde{z}(n))\right) \\ \varphi\left(\tilde{f}(\tilde{z}(n-l_1))\right) \\ \varphi\left(\tilde{f}(\tilde{z}(n-l_2))\right) \end{bmatrix}$$

$$\bar{\eta}_1(n) = \frac{1}{l_1+1} \sum_{i=n-l_1}^{n} \varphi(\tilde{z}(i)), \qquad \bar{\eta}_2(n) = \frac{1}{l(n) - l_1 + 1} \sum_{i=n-l(n)}^{n-l_1} \varphi(\tilde{z}(i))$$

And

$$\bar{\eta}_3(n) = \frac{1}{l_2 - l(n) + 1} \sum_{i=n-l_2}^{n-l(n)} \varphi(\tilde{z}(i))$$

Calculating the $\Delta\, v_i(n) (i = 1,2,3)$ note that

$$\Delta V(n) \le \bar{\zeta}^T(n) \bar{\phi}(l(n)) \bar{\zeta}(n)$$

Hence, system (9) is asymptotically stable if $\phi(l(n)) < 0$ for all $l(n) \in [l_1, l_2]$. Since $\bar{\phi}(l(n))$ is affine with respect to $l(n)$, $\bar{\phi}(l(n)) < 0$ is equivalent to $\bar{\phi}(l_1) < 0$ and $\bar{\phi}(l_2) < 0$. Then, system (9) is asymptotically stable if the two conditions hold.

Theorem 2

Suppose $(H_1), (H_2)$ and the following conditions are satisfied

$$\tilde{\alpha} - \tilde{\beta}(1-\tilde{\alpha})^{-\tau} > 0, \sum_{v=1}^{n-1} e^{-(n-v)} < m, 0 < \tilde{\alpha} < 1$$

Where

$$\breve{\alpha} = 1 - \left\|\left(\varphi(E) - \varphi(\theta(h))\varphi(D)\right)\right\|, \breve{\beta} = \|\varphi(\theta(h))\|\left(\|\bar{\varphi}(\bar{A})\| + \|\bar{\varphi}(\bar{B})\|\right)\|L\|$$

and $\delta = \|\varphi(\theta(h))\|\left(\|\bar{\varphi}(\bar{A})\| + \|\bar{\varphi}(\bar{B})\|\right)\|N\|, \mathcal{K} = \alpha - \beta(1-\alpha)^{-\tau}$ Then the $\varphi(\tilde{z}(n))$ of systems (10) is global attractivity.

Proof

To see this , let us set

$$\tilde{z}(n) = z(n) - z*$$

Where z^* is an equilibrium point of the system (10) if and only if

$$-\varphi(\theta(h))\underline{\varphi}(D)\varphi(z^*) + \varphi(\theta(h))(\varphi(B))\varphi(\gamma^*) + \varphi(\theta(h))\varphi(u) = 0$$

And $\varphi(\gamma^*) \in \overline{C}0[\varphi(f(z^*))]$. It follows from the above that the system (10) can be transformed to

$$\varphi(\tilde{z}(n+1)) = (\varphi(E) - \varphi(\theta(h))\varphi(D)\varphi(\tilde{z}(n)) + \varphi(\theta(h))\varphi(A)\varphi(\tilde{\gamma}(n)) + \varphi(\theta(h))\varphi(B)\varphi\left(\tilde{\gamma}(n-l(n))\right)$$

Where $\varphi(\tilde{\gamma}(.)) = \varphi(\gamma(.)) - \varphi(\gamma^*)$

Since the condition (H_2), note that

$$\begin{aligned} \|\varphi(\tilde{z}(n+1))\| \le{} & \left\|\left(\varphi(E) - \varphi(\theta(h))\varphi(D)\right)\right\| \|\varphi(\tilde{z}(n))\| \\ & + \|\varphi(\theta(h))\|(\|\bar{\varphi}(\bar{A})\| + \|\bar{\varphi}(\bar{B})\|)\|L\|\|\varphi(\tilde{z}_n)\|c \\ & + \|\varphi(\theta(h))\|(\|\bar{\varphi}(\bar{A})\| + \|\bar{\varphi}(\bar{B})\|)\|N\| \end{aligned}$$

Clearly

$$\|\varphi(\tilde{z}(n))\| \le \|\varphi(\tilde{y}(n))\|$$

Under the same initial conditions if $\varphi(y(n))$ satisfies the following equation

$$\|\varphi(\tilde{y}(n+1))\| = \left\|\left(\varphi(E) - \varphi(\theta(h))\varphi(D)\right)\right\| \|\varphi(\tilde{y}(n))\|$$
$$+\|\varphi(\theta(h))\|(\|\overline{\varphi}(\overline{A})\| + \|\overline{\varphi}(\overline{B})\|)\|L\|\|\varphi(\overline{y}_n)\|c$$
$$+\|\varphi(\theta(h))\|(\|\varphi(A)\| + \|\bar{\varphi}(B)\|)\|N\|$$
$$= (1-\breve{\alpha})\|\varphi(\tilde{y}(n))\| + \breve{\beta}\|\varphi(\tilde{y}_n)\|c + \delta$$

In fact, it shows that

$$\|\varphi(\tilde{y}(n+1))\| \geq (1-\breve{\alpha})\|\varphi(\tilde{y}(n))\|c\,,\ \|\varphi(\tilde{y}_n)\|c \leq (1-\breve{\alpha})^{-\tau}$$

Overall, We have

$$\|\varphi(\tilde{y}(n))\| \leq [1-\tilde{\alpha}+\tilde{\beta}(1-\tilde{\alpha})^{-\tau})\, \|\varphi(\tilde{y}(n-1))\| + \delta$$
$$\leq \|\varphi(\tilde{y}(0))\|e^{kn} + \sum_{v=1}^{n-1} e^{-(n-v)k} + \delta$$
$$\leq \|\varphi(\tilde{y}(0))\|e^{kn} + m + \delta$$

Where $\kappa = \breve{\alpha} - \breve{\beta}(1-\breve{\alpha})^{-\tau}$

Since the comparison principle, we obtain that

$$\|\varphi(\tilde{z}(n))\| \leq \|\varphi(\tilde{z}(0))\|e^{-kn} + m + \delta$$

Let the set

$$S = \{\varphi(\tilde{z}(n)) : \|\varphi(\tilde{z}(n))\| \leq m + \delta\}$$

Then, the $\varphi(\tilde{z}(n))$ of system (10) is global attractivity and S is a globally attractive set of the equilibrium point $\varphi(\tilde{z}^*)$.

Hence proved

CONCLUSION

In addition, the memristive neural network has larger storage capacities, stronger learning and memory abilities, and better information processing abilities in virtue of combining the advantages of memristor and cross array. Hence, in recent years, the study on the memristive neural network has become a hotspot in many fields.

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