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Some Notions of Regular Fuzzy Graph of Ternary Semigroups

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ABSTRACT

In this paper is to connect fuzzy theory and graph theory with an algebraic structure ternary semi group. In this paper, we introduce the notion of fuzzy graph of ternarysemi group, the notion of isomorphism of fuzzy graphs of ternary semi groups, the notion of regular fuzzy graph of ternary semi group and the notion of anti fuzzy ideal graph of ternary semi group as a generalization of anti fuzzy ideal of ternary semi group. We study some of their properties and prove that $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be fuzzy graphs of ternary semi groups are isomorphic if and only if their complements are isomorphic.

Keywords : Ternary semi group, fuzzy ideal, anti fuzzy ideal, fuzzy graph, fuzzy graph of ternary semi group, anti fuzzy ideal graph, isomorphic fuzzy graphs of ternary semi groups, regular fuzzy graph of ternary semi group.

Introduction

The formal study of semi groups begins in the early 20th century. Semi groups are basic algebraic structures in many braches of engineering like automata, formal languages, coding theory, finite state machines. The majorrole of graph theory in computer applications is the development of graph algorithms. A number of algorithms are used to solve problems that are modeled in the form of graphs. In 1965, Zadeh [24] introduced the fuzzy theory. The aim of this theory is to develop theory which deals with problem of un certainty Zadeh [25] introduced the notion of interval-valued fuzzy sets and Atanassov [3] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh’s fuzzy set for representing vagueness and uncertainty. The concept of fuzzy set was applied to theory of subgroups by Rosenfeld [21]. After that Kuroki [9] and Mordeson et al. [11] studied theory of fuzzy semi groups. Jun et al. [5, 7] studied theory of fuzzy semi groups and fuzzy $\Gamma-$ rings. Murali Krishna Rao [8-16] studied Anti fuzzy kideals and anti homomorphisms of Γ-semiring and fuzzy soft $\Gamma-$ semi rings. The first definition of fuzzy graph was introduced by Kauffman

[9] in 1973 based on Zadeh's fuzzy relations. In 1975, Rosenfeld considered fuzzy relations on fuzzy subsets and developed the theory of fuzzy graphs as a generalization of Euler's graph theory obtaining analogs of several graph theoretical concepts. Rosenfeld introduced fuzzy graph to model real life situations. If there is a vagueness in the description of objects or in its relationships or in both then we need to a assign a fuzzy graph model. Fuzzy graphs are useful to represent relationships which deal with uncertainty. Fuzzy graph theory is useful in solving the combinatorial problems in data structure theory, data mining, neural networks, cluster analysis and etc. Mordeson and Peng [10] defined the concept of complement of fuzzy graph and described some operations on fuzzy graphs. Akram [1, 2] introduced many new concepts including bipolar fuzzy graphs, interval-valued line fuzzy graphs and strong intuitionistic fuzzy graphs. Bhagacharya [4], Sunitha et al.[22]studied fuzzy graphs. In this paper, we introduce the notion of fuzzy graph of ternary semi group, the notion of isomorphism of fuzzy graphs of ternary semi groups, the notion of regular fuzzy graph of ternary semi group and the notion of anti fuzzy ideal graph of ternary semi group as a generalization of anti fuzzy ideal of ternary semi group, fuzzy graph and graph. The main objective of this paper is to connect fuzzy theory and graph theory with algebraic structure. D.Madhusudhana Rao and G.Srinivasa Rao P.Siva Prasad discussed on ternary semi groups and ternary semi rings [25,26].

Literature Review

2.PRELIMINARIES

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

DEFINITION 2.1. A graph is a pair (V, E), where V is a non-empty set and E is a set of unordered pairs of elements of V.

DEFINITION 2.2. A simple graph is an undirected graph without loops and multiple edges.

DEFINITION 2.3. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge.

DEFINITION 2.4. A graph G(V, E) is connected if there exists a path between every two vertices a and b of V.

DEFINITION 2.5. The number of vertices in a graph G(V, E) is called an order of G(V, E) and it is denoted by |V|.

DEFINITION 2.6. The number of edges in a graph G(V, E) is called a size of graph G(V, E)and it is denoted by |E| .

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DEFINITION 2.7. The neighbor set of a vertex x of graph G(V, E) is the set of all elements in V which are adjacent to x and it is denoted by N (x).

DEFINITION 2.8. The degree of vertex x of graph G(V, E) is defined as the number of edges incident on x and it is denoted by d(x) or equivalently deg(x) = |N (x)|. Let G(V, E) be a graph. Then twice the number of edges of graph G(V, E) is sum of the degrees of all vertices belong to V.

DEFINITION2.9. A graph G(V, E) is said to be k −regular graph if deg(v) = k for all v ∈ V.

DEFINITION2.10. Let S be a non-empty set. A mapping f : S $\rightarrow$[0, 1] is called a fuzzy subset of S.

DEFINITION2.11.Let S be a semi group. A fuzzy subset μ of S is said to be fuzzy sub semi group of S if it satisfies μ(xyz) = min{μ(x), μ(y), μ(z)} for all x, y, z ∈ S.

DEFINITION 2.12. A fuzzy subset μ of a semi group S is called a fuzzy left (lateral, right) ideal of S if μ(xyz) = μ(y) μ(z) (μ(x)){ μ(z) (μ(y)), μ(x)μ(y)(μ(z))} for all x, y, z ∈S.

DEFINITION 2.13. A fuzzy subset μ of a semi group S is called a fuzzy ideal of S if μ(xyz) = max{μ(x), μ(y), μ(z)} for all x, y, z ∈ S

DEFINITION2.14. A fuzzy subset μ of a semi group S is called an anti fuzzy ideal of S if μ(xyz)= min{μ(x), μ(y), μ(z)} for all x, y, z ∈ S

DEFINITION 2.15. A map σ : X × X× X → [0, 1] is called a fuzzy relation on a fuzzy subset μ of X if σ(x, y, z)= min{μ(x), μ(y), μ(z)}, for all x, y, z ∈ X. A fuzzy relation σ is symmetric if σ(x, y, z) = σ(z, y, x), for all x, y, z ∈ X. A fuzzy relation σ is reflexive if σ(x, x, x) = μ(x), for all x∈ X.

DEFINITION 2.16. Let V be a non-empty finite set, μ and σ be fuzzy subsets on V and V × V× V respectively. If σ(x, y, z) = min{ μ(x), μ (y), μ(z)}, for all {u, v}, {v, w}, {w, u}∈ E then the pair G = (μ, σ) is called a fuzzy graph over the set V. Here μ and σ are called fuzzy vertex and fuzzy edge of the fuzzy graph G respectively.

DEFINITION 2.17. The underlying crisp graph of a fuzzy graph G = (μ, σ) is denoted by G=(μ*,σ*),whereμ*={x ∈V|μ(x)>0}and σ*={(x,y,z)∈V×V×V|,σ(x,y,z)>0}.

DEFINITION 2.18. A fuzzy graph *G* = (*μ, σ*) is called a strong fuzzy graph if

σ(*x, y, z*) = min{ μ(x),μ(*y*),μ(z)},for all {*u,v*},{*v,w*},{*w,u*}□*E.*

DEFINITION 2.19. A fuzzy graph G = (μ, σ) is called a complete fuzzy graph if σ(x, y, z) =min{μ(x), μ(y), μ(z)}, for all x, y, z ∈V.

DEFINITION 2.20. The order and the size of a fuzzy graph G = (μ, σ) are defined as O(G) = μ(x) and S(G) =σ(x, y, z) respectively.

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DEFINITION 2.21. Let H = (δ, γ) and G = (μ, σ) be fuzzy graphs over the set V. Then H is called a fuzzy ternary sub graph of fuzzy graph G if δ(x) = μ(x) for all x ∈ V and γ(x, y, z) = σ(x, y, z),for all{u,v},{v,w},{w,u}∈ E.

Results

3.FUZZY TERNARY SEMIGROUP

In this section, we introduce the notion of fuzzy graph of ternary semi group as a generalization of fuzzy graph and graph. We study some of their properties. Throughout this paper we will consider only simple graphs with finite number of vertices and edges.

DEFINITION3.1. Let G(V,E) be a graph, (V, .)be a finite commutative semi group and μ be a fuzzy subset of V such that μ(uvw)= min { μ(u), μ(v), μ(w)}for all {u, v}, {v,w}, {w,u} □ E .Then G(V, E) is called a fuzzy graph of ternary semi group. It is denoted by G(V, E, μ) .

DEFINITION 3.2. If G(V, E) be a complete graph. Then fuzzy graph of ternary semi group G(V, E, μ) is called an anti fuzzy ideal graph of ternary semi group V.

REMARK 3.1. Let G(V, E, μ) be an anti fuzzy ideal graph of ternary semi group. Define σ as a fuzzy subset of V × V× V such that σ(x, y, z) = μ(xyz), for all x, y, z □V. Then G = (σ , μ) is a fuzzy graph in the sense of Rosenfeld. Then fuzzy ideal graph G(V, E, μ)is a generalization of anti fuzzy ideal of ternary semi group, fuzzy graph G = (σ, μ) and the graph G(V, E).

DEFINITION 3.3. Let *G*(V, E, μ) be a fuzzy graph of ternary semi group.

1. The order of G(V, E, μ) is defined as

$$\sum_{x \epsilon V} \mu(x)$$

2.The size of G(V, E, μ) is Defined as

$$\sum_{\{x,y\} \epsilon V} \mu(x,y)$$

3. The degree of G(V, E, μ) is defined as

$$\sum_{u \neq v \neq w} \mu(uvw)$$

EXAMPLE 3.4 : Let V={*a*, *b*, *c*}. The ternary operation ‘ .’ on V is defined as

.	*a*	*b*	*c*
a	*a*	*b*	*c*
b	*b*	*c*	*a*

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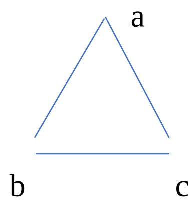
c	*c*	*a*	*b*

Let G(V,E) be a graph Where E={(a,b),(a,c),(b,c)}

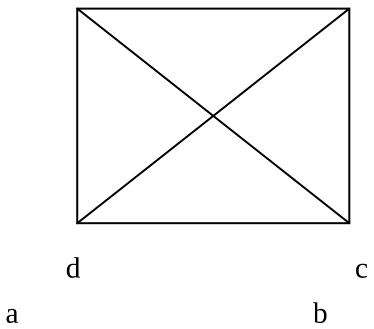


Let μ:V □ [0,1] be a fuzzy subset defined by μ(a)=0.5, μ(b) = 0.4, μ(c)=0.3. By definition 3.1, G(V, E, μ) is a fuzzy graph of ternary semi group V.

EXAMPLE 3.5: Let V = {*a, b, c, d*}. The ternary operation ‘ **.** ‘ on V is defined by

.	*a*	*b*	*c*	*d*
a	*a*	*b*	*c*	*d*
b	*b*	*b*	*c*	*d*
c	*c*	*c*	*c*	*d*
d	*d*	*d*	*d*	*c*

Then (V, **.**) is a commutative ternary semi group. Let G (V, E) be a complete graph where E = {(*a*, *b*), (*b*, *c*), (*c*, *d*), (*d*, *a*), (*d*, *a*), (*d*, *b*)}.



Let μ :V □[0,1] be a fuzzy subset defined by μ (x) = $\begin{cases} 0.5 & if\ x = a \\ 0.2 & if\ x \neq 0 \end{cases}$

Obviously μ is an anti fuzzy ideal of ternary semi group V. By Definition 3.1, G (V, E, μ) is an anti fuzzy ideal graph of ternary semi group.

EXAMPLE 3.6 : Let G (V, E) be a graph with V = {*a*, *b*, *c*} and E= {(*a*, *b*), (*b*, *c*), (*c*, *a*)} and ternary operation ‘ **.** ‘ on V defined by

.	*a*	*b*	*c*
a	*a*	*a*	*c*
b	*a*	*b*	*c*

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c	c	c	c

Obviously (V, .) is a finite commutative ternary semi group.

Define $\mu: V \rightarrow [0,1]$ by $\mu(a) = \frac{1}{2}$, $\mu(b) = \frac{3}{4}$, $\mu(c) = \frac{1}{3}$ Then μ is an anti fuzzy ideal of ternary semi group V. Therefore G(V, E, μ) is an anti fuzzy ideal graph of ternary semi group. Order of an anti fuzzy ideal graph is equal to $\sum \mu(v) = \mu(a) + \mu(b) + \mu(c) = \frac{19}{12}$ Size of an anti fuzzy ideal graph is equal $\sum_{\substack{u \in V, \\ \{u,v\},\{v,w\},\{w.u\} \in E}} \mu(abc) = \mu(abc) + \mu(bca) + \mu(cab) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

DEFINITION 3.7. Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be fuzzy graphs of semi- groups V_1 and V_2, respectively. Then a map $h : V_1 \rightarrow V_2$ such that

i) h is an isomorphism of ternary semi groups

ii) $\mu_1(x) = \mu_2(h(x))$, for all $x \in V_1$

iii) $\mu_1(xyz) = \mu_2(h(x)h(y)h(z))$ for all {x, y}, {y, z}, {z, x} $\in E_1$ and {h(x), h(y)}, {h(y), h(z)}, {h(z), h(x)} $\in E_2$, if and only if h is said to be isomorphism of fuzzy graphs of ternary semi groups. It is denoted by $G(V_1, E_1, \mu_1) \cong G(V_2, E_2, \mu_2)$.

THEOREM 3.8: Let $G(V_1,E_1,\mu_1)$ and $G(V_2,E_2,\mu_2)$ be isomorphic fuzzy graphs of ternary semi group. Then the degree of their vertices are preserved.

Proof. Let h be the isomorphism of fuzzy graphs of ternary semi groups

$G(V_1, E_1,\mu_1)$ and $G(V_2,E_2,\mu_2)$. Then there exists an isomorphism

$h : V_1 \rightarrow V_2$ of ternary semi groups such that $\mu_1(xyz) = \mu_2(h(x)h(y)h(z))$, for all {x, y} $\in E_1$ and {h(x), h(y)} $\in E_2$. Therefore

$$D(u) = \sum_{\substack{u \neq v \neq w, \\ \{u,v\},\{v,w\},\{w,u\} \in E_1}} \mu_1(uvw) = \sum_{\substack{u \neq v \neq w, \\ \{h(u),h(v)\},\{h(v),h(w)\},\{h(w),h(u)\} \in E_2}} \mu_2(h(u)h(v)h(w)) = D(h(u)).$$ Hence the theorem.

THEOREM 3.9: Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphsof ternary semi groups V1 and V2 respectively. Then their orders and sizes are same.

Proof. Suppose $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs of semi groups. Then there exists an isomorphism $h : V_1 \rightarrow V_2$ such that

(i) $\mu_1(x) = \mu_2(h(x))$ for all $x \in V_1$

(ii) $\mu_1(xy) = \mu_2(h(x)h(y))$, for all {x, y} $\in E_1$. and {h(x), h(y)} $\in E_2$

Order of $G(V_1, E_1, \mu_1) = \sum_{v \in V} \mu_1(v) = \sum \mu_2(h(v))$ = Order of $G(V_2, E_2, \mu_2)$.

Size of $G(V_1,E_1,\mu_1) = \sum \mu_1(uvw)$

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{u,v},{v,w},{w,u} □ E_1

= □ □$_2$(h(uvw))

{u,v},{v,w},{w,u} □ E_1

=□ □$_2$(h(u)h(v)h(w))

{h(u),h(v)},{h(v),h(w)},{h(w),h(u)} □ E_2

= Size of $G(V_2, E_2, \mu_2)$.

Hence the theorem.

THEOREM 3.10: Let G(V, E, μ) be a fuzzy graph of ternary semi group. Then

$$\sum_{v\in V} D(v) \leq \sum_{v\in V} d(u)\mu(v)\delta(w)$$

PROOF: Let $v_1, v_2, \ldots, v_n$ be the vertices of fuzzy graph of ternary semi group G(V, E, μ).Then

D(vi) =□ □$_1$($v_i v_j v_k$)□ d(v_i)□ □v_j□□ □v_k□.Hence

v_i □ v_j □ v_k ,{v_i ,v_j },{v_j ,v_k },{v_k ,v_i }□ E

□ D(v_i)□ □ d(v_i)□ □v_j□□ □v_k□.

Hence the theorem.

COROLLARY 3.11 : Let G(V, E, μ) be a fuzzy graph of ternary semi group. Then

□ D□v_i□ □ d□v_i□□ □ □v_j□□ □v_k□. v_i □ V

DEFINITION 3.12 : Let G(V, E, μ) be a fuzzy graph of ternary semi group. Then the complement of G(V,E,μ) is defined as G(V,E,$\bar{\mu}$)

where$\bar{\mu}$ (xyz)=min{μ(x),μ(y)μ(z)}−μ(xyz), for all {x, y}, {y, z}, {z, x} ∈E.

THEOREM 3.13. $G(V_1,E_1,\mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs of ternary semi groups if and only if their complements are isomorphic.

Proof. Suppose that $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs of ternary semi groups Then there exists an isomorphism of ternary semi groups,

h : V_1□V_2such that $\mu_1(x) = \mu_2(h(x))$, for all x ∈V and $\mu_1(xyz) = \mu_2(h(x)h(y)\, h(z))$, {x, y}, {y, z}, {z, x} ∈ E_1, {h(x), h(y)}, {h(y), h(z)}, {h(z), h(x)} ∈ E_2

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$\bar{\mu}_1(xyz)= \min\{\mu_1(x),\mu_1(y),\mu_1(z)-\mu_1(xyz)$

$=\min\{\mu_2(h(x)),\mu_2 h((y)),\mu_2 h((z))-\mu_2(h(x)h(y)\ h(z))$

$=\bar{\mu}_2(h(x)h(y)h(z)),$

for all {x, y}, {y, z}, {z, x}∈ E_1, {h(x), h(y)}, {h(y), h(z)}, {h(z), h(x)∈E_2
Therefore $G(V_1,E_1,\mu_1)$□ $G(V_2,E_2,\mu_2)$.
Similarly we can prove the converse.
Hence the theorem.

THEOREM 3.14 : The complement of complement of fuzzy graph of ternary semi group G(V, E, μ) is itself.

Proof: Suppose the complement of G(V,E,μ)is G(V,E,$\bar{\mu}$)

where$\bar{\mu}$ (xyz)=min{μ(x), μ(y), μ(z)}-$\bar{\mu}$ (xyz) , for all {x, y}, {y, z}, {z, x} ∈E. Then

$\bar{\bar{\mu}}$ (xyz)=min{μ(x), μ(y), μ(z)}-$\bar{\mu}$ (xyz)

= min{μ(x),μ(y),μ(z) }-{min{μ(x),μ(y),μ(z }-μ(xyz)}

= μ(xyz),for all {x,y}, {y,z},{z,x}∈ E Hence the theorem

THEOREM 3.15 : Let G(V, E, μ) be an anti fuzzy ideal graph of ternary semi group. Then$\mu(xyz) = \frac{1}{2}\min\{\mu(x),\mu(y),\mu(z)\}$ for all x,y,z ∈ V if and only if G(V,E,μ)is self-complementary anti fuzzy ideal graph of ternary semi group.

Proof: Let the complement of G(V,E,μ) be G(V,E,$\bar{\mu}$) where

$\bar{\mu}$ (xyz) =min{μ(x),μ(y),μ(z)}- μ(xyz).

Suppose$\mu(xyz) = \frac{1}{2}\min\{\mu(x),\mu(y),\mu(z)\}$ for all x,y,z∈V. Thus 2μ(xyz)min{μ(x),μ(y),μ(z)}and $\bar{\mu}$ (xyz)= μ(xyz), for all x, y, z ∈ V

Therefore $\bar{\mu}$ (xyz)= μ(xyz), for all x, y, z ∈ V. Hence G(V,E,μ) is self-complementary anti fuzzy ideal graph of ternary semi group. Conversely suppose that G(V,E,μ) is self-complementary anti fuzzy ideal graph of ternary semi group where $\bar{\mu}$ (xyz)= min{μ(x),μ(y),μ(z)}- μ(xyz).Then $\bar{\mu}$ (xyz)= μ(xyz), for all x, y, z ∈ V. Thus 2μ(xyz) =min{μ(x),μ(y),μ(z)}and $\mu(xyz) = \frac{1}{2}\min\{\mu(x),\mu(y),\mu(z)\}$ for all x, y, z ∈ V. Hence the theorem.

COROLLARY 3.16: Let G(V, E, μ) be a fuzzy graph of ternary semi group with 0 element and1 element. Then (i) □ □ 0□ □ □ □ u□ , □ u □ V. (ii) □ □ 0□ □ □ □ 1□ .

Proof: Given G(V, E, μ) is a fuzzy graph of ternary semi group with 0 element and 1 element. Then μ(0) = μ(00u) ≤ μ(u) and μ(0) = μ(001) ≤ μ(1). Hence the theorem.

DEFINITION 3.17. Let G(V, E, μ) be a fuzzy graph of ternary semi group . If D(v) = k,for all v ∈V then G(V, E, μ) is said to be regular fuzzy graph of ternary semi group.

DEFINITION 3.18. Let G(V, E, μ) be a fuzzy graph of ternary semi group . Total degree of a vertex u ∈ V is defined as D(u) + μ(u). It is denoted by TD(u).

DEFINITION 3.19. If each vertex of fuzzy graph of ternary semi group G(V, E, μ) has the same total degree k then G(V, E, μ) is said to be totally regular fuzzy graph of semi group of total degree k..

THEOREM 3.20. Let G(V, E, μ) be fuzzy graph of ternary semi group. Then fuzzy subset μ is a constant function if and only if the following are equivalent

i) fuzzy graph of ternary semi group G(V, E, μ) is regular
ii) fuzzy graph of ternary semi group G(V, E, μ) is totally regular.

Proof: Let G(V, E, μ) be a fuzzy graph of ternary semi group and fuzzy subset μbe a constant function.

⇒ (ii) : Suppose fuzzy graph of ternary semi group G(V, E, μ) is regular, D(u) = k, and μ(u) = c. for all u ∈V . TD(u) = D(u) + μ(u) = k + c, for all u ∈V. Hence (i) ⇒(ii).
⇒ (i) : Suppose a fuzzy graph of ternary semi group G(V, E, μ) is totally regular and TD(u) = k, for all u ∈V ⇒ D(u) + μ(u) = k, for all u ∈V ⇒ D(u) + c = k ⇒ D(u) =k − c. Therefore fuzzy graph of semi group G(V, E, μ) is regular. Converse is obvious. Hence the theorem

Conclusion

We introduced the notion of fuzzy graph of ternary semi group, the notion of isomorphism of fuzzy graphs of ternary semi group, the notion of regular fuzzy graph of ternary semi groups and the notion of anti fuzzy ideal graph of ternary semi group as a generalization of anti fuzzy ideal of ternary semi group, fuzzy graph and graph. We studied some of their properties.

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