Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

Impact of Diffusion on a Special Ecosystem

***K.V.L.N.Acharyulu[1],G.Basava Kumar[2], I.Pothuraju[3] & S.V.Vasavi[4]**
[1,2,3,4] Faculty, Department of Maths, Bapatla Engineering College (Autonomous)
Bapatla-522102, Andhra Pradesh, India.
(* Corresponding Author*: vlnacharyulu.kanduri@becbapatla.ac.in or kvlna@yahoo.com)*

ABSTRACT

In this paper, the authors aim to discuss a special ecosystem with constant rates of mortality and harvesting for Ammensal Species. Local stability is established at interior equilibrium point. Spatiotemporal Analysis has been carried out. Global stability is identified at the interior point. The impact of diffusion on this model is traced.

Key words:
Ammensal , Enemy Species, Stability, Ruth-Hurwitz(R-H) stability criterion, Diffusion, HPM, Spatiotemporal Analysis

1. Introduction

Biomathematics, also called as mathematical biosciences, is a multidisciplinary field with a wide and exponentially increasing literature distributed across the last century through numerous disciplines. Mathematicians, physicists, computational scientists, ecologists, medical scientists, demographers, several researchers have made contributions to it. In mathematical biosciences, the main aim is to gain an overview and better understanding about the challenges of real life. For the last few decades, Mathematicians research the merits and demerits of mathematical simulation, mathematical techniques. It has been the foundation of the modern growth of scientific research.

Mathematical modelling in the modern sciences of real life situations is an effort in the language of mathematics to define and explain certain instances of everyday life. Although the reach of mathematical modelling is in many sectors expanding and deepening, it is not only confined to the usage of already proven mathematical techniques. It is very important to notice that one of the main tasks of mathematicians employed in fields such as life, medical and social sciences is to create new mathematical strategies that deal with complicated problems that exist in nature and in our routine as well. Situations are also very difficult in life sciences. As such, before attempting to devise a new mathematical model, one must have some experience regarding the scenario. If a model is formulated, by using an appropriate mathematical technique, the effects may be noticed and the findings are contrasted. Further modifications to the model are indicated by the inconsistencies between theoretical assumptions centered on the model and real life observations.

K.V.L.N.Acharyulu and N.Ch.Pattabhi Ramacharyulu examined different cases of ecological Ammensal models [5-18] and also investigated various ecological models for their stability in manifold dimensions. Many Research scholars [1-4] and Mathematicians[19-31] extended their significant contributions in this modelling field.

A real life problem may seldom be converted into a mathematical problem with all its generality. Even though it can be converted in such a way, it might not be feasible to satisfactorily solve the resulting mathematical problem. Therefore, it will be appropriate to 'simplify' or 'idealize' or 'approximate' the problem with another problem, taken as a relevant model of the initial problem and mathematically interpreted and solved at the same time. All the basic characteristics of the issue will be kept in this

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

idealisation phase, giving up those characteristics that are not inevitable or so relevant to the condition under investigation.

1.1 Notations:
This is an evolutionary environment where Ammensal and Enemy species live together. It is believed that all interacting ecological species are continuously harvested (migrated or immigrated) by depending upon available natural resources with constant mortality rate. Here
(i).X is the density of Ammensal species with natural growth rate a_1,
(ii).Y is the density of the enemy species with natural growth rate a_2,
(iii).$h_1=a_{11}H_1$ is the harvesting of Ammensal species,
(iv). $K_i = \frac{a_i}{a_{ii}}$ be the carrying capacity of Ammensal Species .
(v). $\alpha = \frac{a_{12}}{a_{11}}$ be Ammensalism's coefficient.
(vi).m= Decrease of Ammensal species due to harvesting.
Assume that the parameters described above are positive.

2. Constriction of Mathematical Model

The equations for the special ecosystem are presented as below:
The rate of the growth for Ammensal Species with constant rates of mortality and harvesting

$$\frac{dX}{dt} = a_{11}(-K_1X - X^2 - \alpha XY + H_1) \qquad (2.1)$$

Equation for the growth rate of the Enemy species

$$\frac{dY}{dt} = a_{22}Y(K_2 - Y) \qquad (2.2)$$

2.1. Normal State:
Here $E_1(0,0)$, $E_2(x^\theta,0)$, $E_3(0,y^\varphi)$, $E_4(x^*,y^*)$ are eqilibrium points
The stability of the system around the internal equilibrium $E_4(x^*, y^*)$ point is being studied

only where $x^* = \frac{\sqrt{(K_1+\alpha K_2)^2 + 4H_1} - (K_1+\alpha K_2)}{2}$, $y^* = K_2$ (Co-existence state)

3. Local Stability $E_4\left(x^*, y^*\right)$ by Routh-Hurwitz Stability Criterion

Local stability at equilibrium point implies that if you bring the system next to the point anywhere, it can shift itself in any period of time to the equilibrium point. Global equilibrium implies that from every possible starting stage, the system would arrive at the equilibrium point. In general, Routh-Hurwitz stability criterion is the best tool to discuss the Local stability and Lyapunov Theorem is an effective method to establish Global Stability.
We work out the variational matrix about

$$J = \begin{bmatrix} -a_{11}(k_1 + 2x^* + \alpha y^*) & -\alpha a_{11}x^* \\ 0 & a_{22}(k_2 - 2y^*) \end{bmatrix}$$

$$\Rightarrow J = \begin{bmatrix} -a_{11}(x^* + \frac{H_1}{x^*}) & -\alpha a_{11}x^* \\ 0 & -a_{22}(y^*) \end{bmatrix}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

Characteristic equation of J at E_4 is $\begin{vmatrix} -a_{11}(x^* + \frac{H_1}{x^*}) - \lambda & -\alpha a_{11} x^* \\ 0 & -a_{22}(y^*) - \lambda \end{vmatrix} = 0$

$$\Rightarrow \lambda^2 + \lambda\left[a_{22}y^* + a_{11}\left(\frac{H_1 + x}{x^*}\right)\right] + a_{11}a_{22}\left(\frac{H_1 + x^{*2}}{x^*}\right) = 0$$

Let $A = a_{22}y^* + a_{11}\left(\frac{H_1 + x}{x^*}\right)$ $B = a_{11}a_{22}\left(\frac{H_1 + x^{*2}}{x^*}\right)$

By using R-H stability criteria,

Clearly $A = a_{22}y^* + a_{11}\left(\frac{H_1 + x}{x^*}\right) > 0$ and $B = a_{11}a_{22}\left(\frac{H_1 + x^{*2}}{x^*}\right) > 0$

All the elements (coefficients) in the first column of Routh array are positive.

Hence by R-H stability Criteria, it can be stated that the model is locally stable at interior equilibrium point

$$\left(\frac{\sqrt{(K_1 + \alpha K_2)^2 + 4H_1} - (K_1 + \alpha K_2)}{2}, K_2\right) \quad i.e \;\; E_4(x^*, y^*)$$

4. Spatiotemporal Analysis

Due to the development and implementation of innovative mathematical tools enabling the analysis of broad spatiotemporal databases, spatiotemporal data analysis is emerging the research fields. Spatiotemporal models emerge as data is gathered over time and space, with at least one spatial and one temporal property. An occurrence in a spatiotemporal dataset represents a spatial and temporal process that occurs at a time t and x. Other uses for spatiotemporal research cover genetics, geography, meteorology, medicine, and including transport

Spatiotemporal data processing involves all temporal and spatial similarities are taken into considera tion.Evaluation of the time and space measurements of data bringstremendous uncertainty to the data proces sing method .In such complicated computational situations. Spatiotemporal Analysis can help us to analize the models.

Here we considered an ecological system where Ammensal and enemy are living together. It is assumed that Ammensal species is harvested with constant mortality rate at a constant rate.

Let us consider the diffusive equation system as

$$\frac{\partial X}{\partial t} = a_{11}(-K_1 X - X^2 - \alpha XY + H_1) + D_1 \frac{\partial^2 X}{\partial s^2} \tag{4.1}$$

$$\frac{\partial Y}{\partial t} = a_{22}Y(K_2 - Y) + D_2 \frac{\partial^2 Y}{\partial s^2} \tag{4.2}$$

In this D_1, D_2 represent the constant diffusion coefficients of the Ammensal & Enemy.

The set of equations (4.1)-(4.2) is a diffusion system with the conditions on $X(s,t), Y(s,t)$ in $0 \le u \le L, L > 0$ and

$$\frac{\partial X(0,t)}{\partial t} = \frac{\partial X(L,t)}{\partial t} = \frac{\partial Y(0,t)}{\partial t} = \frac{\partial Y(L,t)}{\partial t} = 0 \tag{4.3}$$

The system (4.1)-(4.3) can be linearized by positioning the system to address its steady condition. In terms of the inner steady state, the linear component of the system (4.1) - (4.2) is accomplished as

$$\frac{\partial X}{\partial t} = -a_{11}Xx^* - a_{11}\alpha Yx^* + D_1 \frac{\partial^2 X}{\partial s^2} \tag{4.4}$$

$$\frac{\partial Y}{\partial t} = -a_{22}y^*Y + D_2 \frac{\partial^2 Y}{\partial s^2} \tag{4.5}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

by assuming $x = x^* + X$ and $y = y^* + Y$. Let the solution of the system (4.4)-(4.5) be the form $X(s,t) = \alpha_1 e^{\lambda t} \cos ku$, $Y(s,t) = \alpha_1 e^{\lambda t} \cos ku$; where α_1 , α_2 are amplitudes and k is the wave number of the solution. X , Y are propagations of populations. Corresponding to the diffusive system (4.4)-(4.5), the characteristic equation is

$$\mu^2 + A\mu + B = 0 \tag{4.6}$$

where $A = a_{11}x^* + a_{22}y^* + k^2(D_1 + D_2)$; $B = a_{11}a_{22}x^* y^* + k^2(a_{11}D_1x^* + a_{22}D_2y^*) + D_1D_2k^4$.

Theorem (4.1): The interior equilibrium is locally asymptotically stable in the presence of diffusion if and only if $A > 0$ and $B > 0$.
This theorem follows immediately by Routh-Hurwitz criteria.

$$A = a_{11}x^* + a_{22}y^* + k^2(D_1 + D_2) > 0 \quad B = (a_{11}a_{22}x^* y^* + k^2(a_{11}D_1x^* + a_{22}D_2y^*) + D_1D_2k^4) > 0$$

Theorem (4.2):
If the system at the interior equilibrium point is globally stable without diffusion.Also under zero flux boundary conditions, the resulting diffusive model (4.1)-(4.3) is asymptotically stable globally by significantly enhancing the coefficients of diffusion.

Proof: Let us define the function $V_1(t) = \int_0^R V(X,Y) ds$,

Now we differentiate V_1 w.r.to t along with X and Y

$$\frac{dV_1}{dt} = \int_0^R \left(\frac{\partial V}{\partial X}\frac{\partial X}{\partial t} + \frac{\partial V}{\partial Y}\frac{\partial Y}{\partial t} \right) ds = I_1 + I_2 \tag{4.7}$$

where $I_1 = \int_0^R \frac{dV}{dt} dx$ and $I_2 = \int_0^R \left(D_1 \frac{\partial V}{\partial X}\frac{\partial^2 X}{\partial s^2} + D_2 \frac{\partial V}{\partial X}\frac{\partial^2 X}{\partial s^2} \right) ds$ (4.8)

Using the established result of B.Dubey & J.Hussain [2],

we get $I_2 = -D_1 \int_0^R \frac{\partial^2 V}{\partial X^2} \left(\frac{\partial X}{\partial s} \right)^2 ds - D_2 \int_0^R \frac{\partial^2 V}{\partial Y^2} \left(\frac{\partial Y}{\partial s} \right)^2 ds$

$$= -D_1 \int_0^R \frac{X^*}{X^2} \left(\frac{\partial X}{\partial s} \right)^2 ds - D_2 \int_0^R \frac{Y^*}{Y^2} \left(\frac{\partial Y}{\partial s} \right)^2 ds \tag{4.9}$$

From (4.7), (4.8) and (4.9),

Clearly $I_1 < 0$ then $V_1'(t)$ is negative, it could easily be observed.If $I_1 < 0$, then it can be observed that by enhancing the sufficiently large diffusion coefficients D_1 and D_2, $V_1'(t)$ can be made negative.

5. Global Stability of the Special Ecosystem

Now we discuss the global stability of the considered Special Ecosystem in which the Ammensal Species is influenced by constant rate of mortality and harvesting at variable rate.
The equations are
(i).The rate of the growth for Ammensal Species

$$\frac{dX}{dt} = a_{11}(-(1-m) K_1 X - X^2 - \alpha X Y + H_1) \tag{5.1}$$

(ii).Equation for the growth rate of the Enemy species

$$\frac{dY}{dt} = a_{22}Y(K_2 - Y) \tag{5.2}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

By taking perturbations(P_1,P_2) and after linearization, we get

$$\frac{dP_1}{dt} = -\left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11}P_1 - \alpha a_{11} P_2 \overline{X}$$

$$\frac{dP_2}{dt} = -a_{22} P_2 \overline{Y}$$

The corresponding characteristic equation is

$$\lambda^2 + \left[\left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11} + a_{22}\overline{Y}\right]\lambda + \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11}a_{22}\overline{Y} = 0$$

The above equation is in the form $\lambda^2 + A\lambda + B = 0$

where $A = \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2\,a_{11} + a_{22}\overline{Y} > 0$ (5.3)

$$B = \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] \overline{Y}\; 2a_{11}a_{22} > 0 \quad (5.4)$$

The criteria for the presence of the function of Liapunov are fulfilled.

Now, we define $E(P_1, P_2) = \frac{1}{2}\left(R P_1^2 + 2 S P_1 P_2 + T P_2^2\right)$ (5.5)

where $R = \dfrac{\left(a_{22}\overline{X}\right)^2 + \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11}a_{22}\overline{Y}}{D}$ (5.6)

$$S = -\frac{\alpha\, a_{11}\, a_{22}\, \overline{X}\,\overline{Y}}{D} \quad (5.7)$$

$$T = \frac{\left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right]^2 4a_{11}^2 + \alpha^2 a_{11}^2 \overline{X}^2 + \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11}\, a_{22}\, \overline{Y}}{D} \quad (5.8)$$

and $D = AB > 0$

It is obvious from the equations (5.3) and (5.4) that D>0 and R>0.

$$Also\, D^2\left(RS - T^2\right) = D^2 \left\{ \left(\frac{a_{22}^2 \overline{Y}^2 + 2a_{11}\, a_{22}\, \overline{Y}\left[X + \left(\frac{(1-m)K_1 + \alpha\, K_2}{2}\right)\right]}{D} \right) \right.$$

$$\left(\frac{\left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right]^2 4a_{11}^2 + \left(\alpha a_{11}\overline{X}\right)^2 + \left[\overline{X} + \left(\frac{(1-m)K_1 + \alpha K_2}{2}\right)\right] 2a_{11}a_{22}\overline{Y}}{D} \right)$$

$$\left. - \frac{\left(\alpha a_{11} a_{22} \overline{X}\,\overline{Y}\right)^2}{D^2} \right\}$$

$$\Rightarrow D^2\left(RT - S^2\right) > 0 \Rightarrow RT - S^2 > 0 \;\; i.e\; S^2 - RT < 0$$

The function E(P_1,P_2) at (5.5) is positive definite.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 1, 2021, Pages. 1845 - 1852
Received 15 December 2020; Accepted 05 January 2021.

$$\therefore \frac{\partial V}{\partial P_1}\frac{dP_1}{dt}+\frac{\partial V}{\partial P_2}\frac{dP_2}{dt}=-\left(P_1^2+P_2^2\right)<0$$

This is obviously a negative definite.
So, $E(P_1,P_2)$ is a Liapunov's function for the linear system.
Next we prove that $E(P_1,P_2)$ is also a Liapunov's function for the non-linear system.
Let γ_1 and γ_2 be two functions of X and Y defined by

$$\gamma_1(X,Y)=a_{11}\left(-(1-m)K_1X-X^2-\alpha X\,Y+H_1\right)$$

$$\gamma_2(X,Y)=a_{22}Y\left[K_2-Y\right]$$

We must now prove that $\frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2$ is certainly negative.

By taking Perturbations $X=\overline{X}+P_1$ and $Y=\overline{Y}+P_2$ in the above equations, weget

$$\gamma_1(P_1,P_2)=\frac{dP_1}{dt}=-2a_{11}\overline{X}+\left(\frac{(1-m)K_1+\alpha K_2}{2}\right)P_1-a_{11}\alpha\overline{X}\,P_2+\beta_1\left(P_1,P_2\right)$$

where $\beta_1\left(P_1,P_2\right)=-a_{11}P_1^2-a_{11}\alpha P_1P_2$

Similarly

$$\gamma_2(P_1,P_2)=\frac{dU_2}{dt}=-a_{22}\overline{Y}\,P_2+\beta_2\left(P_1,P_2\right)\text{where }\ \beta_2\left(P_1,P_2\right)=-a_{22}P_2^{\,2}$$

From (5.5) $\frac{\partial V}{\partial P_1}=RU_1+SU_2$ and $\frac{\partial V}{\partial P_2}=S\,U_1+T\,U_2$

$$\text{Now } \frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2=\left(RP_1+SP_2\right)\left(-2a_{11}\overline{X}+\left(\frac{(1-m)K_1+\alpha K_2}{2}\right)P_1-a_{11}\alpha\overline{X}\,P_2+\beta_1\left(P_1,P_2\right)\right)$$

$$+\left(S\,P_1+T\,P_2\right)\left(-a_{22}\,\overline{Y}\,P_2+\beta_2\left(P_1,P_2\right)\right)$$

$$\text{Now } \frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2=-P_1^2+P_2^{\,2}+\left(R\,P_1+S\,P_2\right)\beta_1(P_1,P_2)+\left(S\,P_1+T\,P_2\right)\beta_2(P_1,P_2)$$

Introducing polar coordinates $P_1=\mu\cos\theta$ & $P_2=\mu\sin\theta$

$$\frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2=-\mu^2+\mu[(R\cos\phi+S\sin\phi)\beta_1\left(P_1,P_2\right)+(S\cos\phi+T\sin\phi)\,\beta_2\left(P_1,P_2\right)]$$

Let us denote the largest of the numbers $|R|$,$|S|$,$|T|$ *by* λ

$$\left|\beta_1\left(P_1,P_2\right)\right|<\frac{\mu}{6\lambda}\ \text{ and }\ \left|\beta_2\left(P_1,P_2\right)\right|<\frac{\mu}{6\lambda}$$

for all sufficiently small $\mu>0$, $\frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2<\ -\mu^2+\frac{4\lambda\mu^2}{6\lambda}=-\frac{\mu^2}{3}<0$

E is thus a positive definite function according to the condition that
$\frac{\partial V}{\partial P_1}\gamma_1+\frac{\partial V}{\partial P_2}\gamma_2$ is negative definite Thus the equilibrium state is "asymptotically stable" globally.

6. Conclusions

A research was extensively carried out on a special model of Mortal Ammensal and Enemy species. The following conclusions are put forth:
(i). Local stability is noticed by the Routh-Hurwitz criteria at the interior equilibrium point
(v). Diffusion analysis addresses the system's stability.

(vi).Global stability is established at interior Point.

Acknowledgments

The authors are grateful to **Dr.*Komaravolu Chandrasekharan***, A great and renounced mathematician from Bapatla, India for his unforgettable and most valuable contributions in the field of Mathematical Research The authors are also thankful to **Dr.V.Damodara Naidu**, Principal, Bapatla Engineering College, Bapatla and **the Management, Bapatla Education Society** for their constant encouragement and valuable support.

References

[1]. B.Dubey and J.Hussian (2000), Modelling the interaction of two biological species in a polluted environment, J. Math. Anal. Appl., 246,58–79.

[2]. D.Del-Castillo-Negrete, B.A. Carreras, V. Lynch (2002),Front propagation and segregation in a reaction–diffusion model with cross-diffusion, Phys. D 168, 45–60.

[3]. G.C. Cruywagen, J.D. Murray, P.K. Maini (1997), Biological pattern formation on two-dimensional spatial domains: a nonlinear bifurcation analysis, SIAM J. Appl. Math. 57 ,1485–1509.

[4]. J.D. Murray (1993), Mathematical Biology, Springer-Verlag, Berlin.

[5]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2010), AnAmmensal-Enemy Specie Pair With Limited And Unlimited Resources Respectively-A Numerical Approach, Int. J. Open Problems Compt. Math (IJOPCM), 3(1),73-91.

[6]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2010), An Enemy- Ammensal Species Pair With Limited Resources –A Numerical Study, Int. J. Open Problems Compt. Math (IJOPCM), 3(3), 339-356.

[7]. K.V.L.N.Acharyulu and N.Ch. PattabhiRamacharyulu (2011), Mortal Ammensal and an Enemy Ecological Model with Immigration for Ammensal Species at a Constant Rate, International Journal of Bio-Science and Bio-Technology,3(1),39-48.

[8]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2011),An Immigrated Ecological Ammensalism with Limited Resources"- International Journal of Advanced Science and Technology,27(1), 87-92.

[9]. K.V.L.N.Acharyulu and N.Ch.Pattabhi Ramacharyulu (2011), A Numerical Study on an Ammensal - Enemy Species Pair with Unlimited Resources and Mortality Rate for Enemy Species"- International Journal of Advanced Science & Technology,30,13-24.

[10]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2011), An Ecological Ammensalism with Multifarious restraints- A Numerical Study" International Journal of Bio-Science and Bio-Technology, 3(2),1-12.

[11]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2011), Multiple Constraints in Ecological Ammensalism- A Numerical Approach, Int. J. Advance. Soft Comput. Appl., 3(2),1-15.

[12]. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu (2012),On the Carrying capacity of Enemy Species, Inhibition coefficient of Ammensal Species and Dominance reversal time in An Ecological Ammensalism - A Special case study with Numerical approach, International Journal of Advanced Science and Technology, 43,49-58.

[13]. K.V.L.N.Acharyulu , N.Rama Gopal ,P.Prasanna Anjaneyulu and P. Rama Mohan (2012), Global Stability Analysis of A Three Level Ecological Ammensalism with Four Species, International Journal of Mathematical Sciences, Technology and Humanities,1(2),838-847.

[14]. K.V.L.N.Acharyulu, N.Phani Kumar, G.Bhargavi & K.Nagamani, Ecological Harvested Ammensal Model- A Homotopy Analysis, International Journal of Scientific and Innovative Mathematical Research,3(12),27-35.

[15]. K.V.L.N.Acharyulu & N.Phani Kumar (2015), Ecological Ammensalism-A Series Solution By Homotopy Perturbation Method, Acta Ciencia Indica, 41(4), 295-304.

[16]. K.V.L.N.Acharyulu, S.V.Vasavi & SK.Khamar Jahan (2016), A Homotopy Series Solution For A Competitive Ecological Model With Harvesting For Second Species, *Acta Ciencia Indica*, 42(3), 207-216.

[17]. K.V.L.N.Acharyulu (2017),Homotopy Analysis on Harvested Enemy Species with Variable Rate and Cover Protected Ammensal Species", Acta Ciencia Indica, 43(3),205-213.

[18]. K.Kalyani, N. SeshagiriRao &K.V.L.N.Acharyulu (2014), A fixed point of self mapping in Hilbert space, Global Journal of Pure and Applied Mathematics , 10(6),783-786.

[19]. Lotka A.J.(1925), Elements of MathematicalBiology, Williams and Wikkins, Baltimore, USA.

[20]. M. Lakshmi Sailaja, K.V.L.N.Acharyulu, N.RamaGopal and P.Rama Mohan (2010), Ecological Ammensal Model With Reserve for One Species and Harvesting both The Species at Variable Rates, International Journal of Advances in Soft Computing and Its Applications, 3(3),264-281.

[21]. Nisbet RM, Gurney WSC.(1982), Modelling Fluctuating Populations. New York : John Wiley.

[22]. N.SeshagiriRao, K.Kalyani and K.V.L.N.Acharyulu (2014),Threshold results for host –Mortal Commensal ecosystem with limited resources, Global Journal of Pure and Applied Mathematics,10(6),787-791.

[23]. N.SeshagiriRao, K.Kalyani and K.V.L.N.Acharyulu (2014), Host-Monad Ecological Model With Phase Plane Analysis, International Journal of Applied Engineering Research, 9(23),21605-21610.

[24]. N.Seshagiri Rao, K.Kalyani and K.V.L.N.Acharyulu (2015), Threshold results of a Host -Mortal commensal ecosystem with a constant harvesting of the Commensal species, ARPN Journal of Engineering and Applied Sciences,10(2),802-805.

[25]. N.SeshagiriRao, K.V.L.N.Acharyulu and K.Kalyani (2015), Numerical study on ecological commensalism between two species with harvested Commensal, ARPN Journal of Engineering and Applied Sciences, 10(4),1548-1551.

[26]. N.Seshagiri Rao, K.V.L.N.Acharyulu & K.Kalyani (2015) , A Host-Mortal Commensal Species Pair With Limited Resources - A Numerical Study, International Journal of Bio -Science and Bio - Technology, 7(1),133-140.

[27]. N.Seshagiri Rao, K.V.L.N.Acharyulu and K.Kalyani(2015) , Phase Plane Analysis of A Host -Commensal Ecological Model, International Journal of Advanced Science and Technology,78, 59-66.

[28]. N.Seshagiri Rao, K.V.L.N.Acharyulu and K.Kalyani (2015), Chaotic behavior of host- Monad commensal species pair- A special numerical case study, ARPN Journal of Engineering and Applied Sciences, 10(14), 5976-5983.

[29]. N.Seshagiri Rao, K.V.L.N.Acharyulu & K.Kalyani (2015), A Host - Mortal Commensal Ecosystem With Harvesting Of Both The Species - Phase Plane Analysis, communicated to ARPN Journal of Engineering and Applied Sciences, 10(13),5490-5497.

[30]. N.Seshagiri Rao, K.V.L.N.Acharyulu and K.Kalyani (2015), Null Clines, Phase Planes Of Both Harvested Host-Mortal Commensal Eco-System, ARPN Journal of Engineering and Applied Sciences,10(14),5976-5983.

[31]. N.Phani Kumar, K.V.L.N.Acharyulu, S.V.Vasavi & SK.KhamarJahan (2015), A Series Solution of Ecological Harvested Commensal Model by Homotopy Perturbation Method, International Journal of Scientific and Innovative Mathematical Research,3(11),44-53.