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Uncertain Multi –Objective Multi –Item Four Dimensional Fractional Transportation Model

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Abstract: In day to day real-life situations we often tend to face problems which contain parameters which are mutually non-commensurable. In transportation problems where goods are transported from origin to destination, when trying to establish a relationship between parameters, we observe that few of them come out as fractions with linear nature. In this work, we have proposed a four dimensional multi-objective multi-item fractional transportation model whose parameters in both objective functions and constraints are considered as an uncertain variables. As it is difficult to handle such type of problem directly, an equivalent deterministic model is obtained by using chance constraint and expected value property of uncertain variable. Furthermore, all the uncertain linear fractional objective functions of the proposed model are converted to a single objective function and obtained an optimal solution for the proposed model. An illustrative example is given to demonstrate the proposed model.

Key words: Multi objective,uncertain transportation problem, fractional linear programming.

1. Introduction

The transportation problem, introduced by Hitchock [1] in 1941, deals with transporting and distributing goods from producer to customer. The original transportation problem involves single objective whereas real world situations are quite complex and many objectives have to be considered. These problems are known as multi-objective transportation problems [MOTP]. Several authors like Lee,Moore,Waiel,Pramanik and Roy[2,3,4] handled and developed solving techniques of MOTPs. The generalization of transportation problems wherein three dimensional properties are considered rather than source and destinations is called solid transportation problem[STP] and it was introduced by Haley[5].Some situations might arise in real life wherein we have to optimize the ratio of two linear functions. For example, the ratio of time taken in the travelled route and the preferred route. These type of problems are known as fractional transportation problem [FTP] and were introduced by Swarup [6] in 1966. Gomathi et al. [7]developed a method for solving the linear transportation problem(LFTP) wherein the LFTP was converted into two separate transportation problem and optimal solution of LFTP was obtained by treating the optimal solution of one TP as the basic feasible

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solution of the other TP problem. Transportation problems consisting of multiple objectives including fractional objectives are known as multi-objective linear fractional transportation problem [MOLFTP]. NuranGuzel[8],by solving non-linear fraction with single objective function, as proposed by Dinklbach(1967)[9],developed the solution to multi-objective linear fractional programming problem[MOLFPP].When we consider multiple objectives and products in a four dimensional fractional transportation problem[4DFTP] it becomes four dimensional multi-objective multi-item fractional transportation problems[4DMOMIFTP]. In a real situations, the parameters of 4DMOMIFTPS like availability of demand and unit transportation cost are not exact always due to the uncertainty in human judgment and lack of information which are uncontrollable factors. But these uncertainties can be considered by fuzzy sets given by Zadeh[10]. Multiple researchers while proposing method to solve TP's have used fuzzy numbers to represent the uncertainty in parameters. To solve fully fuzzy MOTP, the Mehar's method was introduced by Gupta et al.[11].The Method of solving fuzzy MOSTP, taking all parameters as fuzzy numbers excluding decision variable was proposed by Ojha et al.[12]. A Fully fuzzy multi objective multi item solid transportation problem [FFMOMISTP] is a multi objective multi item solid transportation problem [MOMISTPs] wherein every parameter is denoted by fuzzy numbers.Deepika rani et al. [13] presented a method to find the optimal compromise solution of FFMOMISTP. In fuzzy environment (intuitionistic L-R type) and vehicle speed, Dipakkumar et al. formulated multi objective multi item transportation problem in four dimensions [14].A novel approach to solve multi-objective linear fractional programming problem proposed by Moumita and De[15] was extended by Dheyab[16] using complementary development method and it was extended by Jain[17]for MOLFP and fuzzy MOLFP. Using fuzzy comparison to convertFMOMISTP to crisp FMOMISTP, Khalifa et al. [18]proposed a solving method.

Since Zadeh's fuzzy set theory[10] could not handle situations consisting incomplete information, we use uncertainty theory founded by Liu [19] to handle such indeterminacies. Nowadays, uncertainty theory is considered as a mathematical branch for modelling belief degrees and has been adopted in many mathematical models like uncertain programming, uncertain logic, uncertain graph, uncertain statistics and uncertain finance[20],[21],[22]. The belief degree of an uncertain event to happen is measured by uncertain measure. To simultaneously deal with uncertainty and randomness, the usage of random uncertain variable and chance measure was also introduced by Liu[23]. Post that, he also presented uncertain random programming to model optimization problems containing more than one random variable. Gao[24], in his paper, newly proposed certain properties based on continuously uncertain measures. Guo etal. [25] studied transportation problem consisting uncertain cost and random supplies. SeyyedMojtabaChasence[26] propose uncertain linear fractional programming problem and also developed few methods to convert uncertain optimization problem as an equivalent crisp problem. Ali Mahmooderad[27] developed a modelling method for uncertain linear fractional transportation problem. Zhou et al. [28] and Zhong et al. [29]proposed compromise programming models and interactive methods for multi –

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objective problems under uncertain environment. Mohanaselvi et al.[30] developed a multiobjective linear fractional transportation problem under uncertain environment. Cui and Sheng[31]introduced an model for dealing with uncertain solid transportation problem. The compromise solution for fixed charge solid transportation problem was found by Yang etal. [32] by using type -2 uncertain optimization methods.

Inspired by the above authors, we have presented a model for multi-objective multi-item four dimensional fractional transportation problem [MOMI4DFTP] under uncertain environment. The objectives of this model are minimizing transportation cost, transportation time and the damage charges. The objectives we have considered are in fractional form in which all the parameters are normal uncertain variables. The uncertain MOMI4DFTP is converted into an equivalent crisp MOMI4DFTP by using the expected value methods. All the uncertain linear fractional objective functions of the proposed model are converted to a single objective function by using the method introduced by Dinkelbach[9]and obtained an optimal solution for the proposed model.

The content of the article is structured as follows: In section 2, we have presented some definitions and theorems of uncertainty theory which are used in the model. Notations are given under section 3. In section 4, the mathematical model of uncertain multi-objectivemulti-itemfourdimensional fractional transportation problem [UMOMI4DFTP] is introduced. Equivalent deterministic models by using expected value method and chance constraint method are given in the sections 5 and 6 respectively. The procedure for solving the UMOMI4DFTP is given in section 7. A numerical example has been given in section 8. The conclusions have been given in section 9.

2. Preliminaries

Some definitions and basic concepts of uncertainty theory, which have been used in the subsequent discussions are introduced below.

Definition 1: [21],[19] Let $\mathcal{L}$ be a σ –algebra of collection of events Λ of a universal set Γ. A set function $\mathcal{M}$ is said to be uncertain measure defined on the σ– algebra where $\mathcal{M}\{\Lambda\}$ indicate the belief degree with which we believe that the event will happens and satisfies the following axioms:

1. Normality Axiom: For the universal set Γ we have $\mathcal{M}\{\Gamma\}=1.$
2. Duality Axiom: For any event Λ we have $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^C\}=1.$
3. Subadditivity Axiom: For every countable sequence of events $\Lambda_1,\Lambda_2,...,$ we have $\mathcal{M}\{\overset{\infty}{\underset{i=1}{U}}\Lambda_i\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$
4. Product Axiom: Let $(\Gamma_i,\mathcal{L}_i,\mathcal{M}_i)$ be uncertainty spaces for i=1,2,3,…. The product uncertain measure is an uncertain measure holds $\mathcal{M}\{\prod_{i=1}^{\infty}\wedge_i\}=\overset{\infty}{\underset{i=1}{\wedge}}\mathcal{M}\{\wedge_i\}$ where $\Lambda_i\in\mathcal{L}_i$ for $i=1,2,3,....,\infty$.

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Definition 2: [19] A function $\xi:(\Gamma,\mathcal{L},\mathcal{M})\to\Re$ is said to be an uncertain variable such that $\{\xi\in B\}=\{\gamma\in\Gamma/\xi(\gamma)\in B\}$ is an event for any Borel set B of real numbers.

Definition 3: [19] An uncertain variable ξ defined on the uncertainty space $(\Gamma,\mathcal{L},\mathcal{M})$ is said to be non- negative if $\mathcal{M}\{\xi<0\}=0$ and positive if $\mathcal{M}\{\xi\le 0\}=0$.

Definition 4: [19] For any real number *x*, the uncertainty distribution $\phi(x)$ of an uncertain variable ξ is defined by $\phi(x)=\mathcal{M}\{\xi\le x\}$.

Definition 5: Let $\phi(x)$ be the regular uncertainty distribution of an uncertain variable ξ Then $\phi^{-1}(\alpha)$ is called inverse uncertainty distribution of ξ and it exists on (0, 1).

Definition 6: [19] The uncertain variable $\xi_i\quad(i=1,2,...,n)$ are said to be independent

if $\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i\in B_i)\right\}=\mathop{\wedge}_{i=1}^{n}\mathcal{M}(\xi_i\in B_i)$ (1)

where $B_i\quad(i=1,2,...n)$ are called Borel sets of real numbers.

Theorem 1: let ξ be an uncertain variable with regular uncertain distribution function ψ .Then its $\alpha-$ optimistic value and $\alpha-$pessimistic values are

$$\xi_{\sup}(\alpha)=\psi^{-1}(1-\alpha),\quad \xi_{\inf}(\alpha)=\psi^{-1}(\alpha). \qquad (2)$$

Theorem 2: [33] The regular uncertainty distributions of independent uncertain variables $\xi_i\quad(i=1,2,3,...,m,...,n)$ are $\phi_i\quad(i=1,2,...,m,...,n)$ respectively. If the function $f(x_1,x_2,...,x_m,...,x_n)$ is strictly increasing and strictly decreasing with respect to $x_1,x_2,...,x_m$ and $x_{m+1},x_{m+2},...,x_n$ respectively then the uncertain variable $\xi=f(\xi_1,\xi_2,...\xi_m,...,\xi_n)$ has an inverse uncertainty distribution
$\psi^{-1}(\alpha)=f(\phi_1^{-1}(\alpha),\phi_2^{-1}(\alpha),...,\phi_m^{-1}(\alpha),\phi_{m+1}^{-1}(1-\alpha),\phi_{m+2}^{-1}(1-\alpha),...,\phi_n^{-1}(1-\alpha))$ (3)

Definition 7: [19] The expected value of uncertain variable ξ is given by

$$E(\xi)=\int_0^{\infty}\mathcal{M}\{\xi\ge x\}dx-\int_{-\infty}^{0}\mathcal{M}\{\xi\le x\}dx \qquad (4)$$

This is valid only if at least one of the integral is finite.

Theorem 3: [34] The regular uncertainty distributions of independent uncertain variables $\xi_i\quad(i=1,2,...,m,...,n)$ are $\phi_i\quad(i=1,2,...,m,...,n)$ respectively. If the function $f(x_1,x_2,..,x_m..,x_n)$ is strictly increasing and strictly decreasing w.r.to $x_1,x_2,...,x_m$ and $x_{m+1},x_{m+2},...,x_n$ respectively, then

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$$E(\xi) = \int_0^1 f(\phi_1^{-1}(\alpha),....\phi_m^{-1}(\alpha),\phi_{m+1}^{-1}(1-\alpha),....\phi_n^{-1}(1-\alpha))d\alpha \tag{5}$$

From the above theorem, we know that

$$E(\xi) = \int_0^1 \phi^{-1}(\alpha)d\alpha \tag{6}$$

where ξ is an uncertain variable with regular uncertainty distribution Φ.

Definition 9: [19] A normal uncertain variable is defined as

$$\Phi(x) = \left(1+exp^{\left(\frac{\pi(\mu-x)}{\sqrt{3}\sigma}\right)}\right)^{-1}, \ x>=0 \tag{7}$$

It is represented symbolically as N(μ,σ); μ, σ $\in R$ with σ > 0. The inverse uncertainty distribution of N(μ,σ) is defined as

$$\Phi^{-1}(\alpha) = \mu + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} \ . \tag{8}$$

The expected value of normal uncertain variable is given by

$$E[\xi] = \mu. \tag{9}$$

3. Nomenclature

The following notations have been considered for constructing the proposed model:

i number of origins

j number of destinations

k number of mode of transport

r number of transportation routes

p number of products

$\tilde{Z}_n$ uncertain objective functions, where n=1,2,…N

$\frac{\tilde{C}_{ijkrp}}{\tilde{D}_{ijkrp}}$ ratio of the unit transportation actual cost and standard cost of p^{th} good from i^{th} origin to j^{th} destination by k^{th} transport via r^{th} road per unit distance.

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$\dfrac{\tilde{T}^{A}_{ijkrp}}{\tilde{T}^{S}_{ijkrp}}$ ratio of actual transportation time to the standard transportation time

$\dfrac{\tilde{D}^{A}_{ijkrp}}{\tilde{D}^{S}_{ijkrp}}$ ratio of actual damage charge to the standard damage charge per unit

$\tilde{a}_{ip}$ quantity of p^{th} good available at i^{th} origin

$\tilde{b}_{jp}$ the demand of the p^{th} good at the j^{th} destination

$\tilde{e}_{k}$ capacity of a single vehicle of k^{th} transport.

4. Mathematical formulation of UMOMI4DFTP

A multi-objective multi-item four dimensional fractional transportation problem under uncertain environment is defined in equation (10).

where P products can be transported from i origins $\tilde{a}_i$ to j destinations $\tilde{b}_j$ by means of $\tilde{e}_k$ conveyances and n objectives are to be minimized. $\dfrac{\tilde{C}_{ijkrp}}{\tilde{D}_{ijkrp}}$ represents the uncertain unit transportation cost of p^{th} item when it is transported from i^{th} origin to j^{th} destination by k^{th} conveyance via r^{th} route.

$$\begin{cases} \text{Min } \tilde{Z}_n = \dfrac{\tilde{N}_n(x)}{\tilde{D}_n(x)} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P}\tilde{C}_{ijkrp}x_{ijkrp}}{\sum_{r=1}\sum_{j=1}\sum_{k=1}\sum_{r=1}\sum_{p=1}\tilde{D}_{ijkrp}x_{ijkrp}} \quad n=1,2,\ldots,N \\ \text{subject to} \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} \le \tilde{a}_{ip} \quad i=1,2,\ldots I,\ p=1,2,\ldots P \\ \sum_{i=1}\sum_{k=1}\sum_{r=1} x_{ijkrp} \ge \tilde{b}_{jp} \quad j=1,2\ldots J,\ p=1,2,\ldots P \\ \sum_{i}\sum_{j}\sum_{r}\sum_{p} x_{ijkrp} \le \tilde{e}_k \\ x_{ijkrp} \ge 0 \forall i,j,k,r,p \end{cases} \tag{10}$$

As we cannot deal uncertain environment directly, the above model with uncertain supplies, demands, costs, and capacities could be converted as equivalent deterministic model by employing the expected value and chance constraint methods.

5. Expected Value Method for UMOMI4DFTP

In this section, we have obtained the equivalent deterministic model for UMOMI4DFTP using expected value method. The deterministic model of (10) is as follows:

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$$\begin{cases} \text{Min } Z_n = E\left[\dfrac{\sum_i\sum_j\sum_k\sum_r\sum_p \tilde{C}_{ijkrp}x_{ijkrp}}{\sum_i\sum_j\sum_k\sum_r\sum_p \tilde{D}_{ijkrp}x_{ijkrp}}\right] \\ \text{subject to} \\ E\left[\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} - \tilde{a}_{ip}\right] \le 0, i = 1,2,...I, p = 1,2,..P \\ E\left[\tilde{b}_{jp} - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp}\right] \le 0, j = 1,2,...J, p = 1,2,..P \\ E\left[\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P} x_{ijkrp} - \tilde{e}_k\right] \le 0, k = 1,2,...K \\ x_{ijkrp} \ge 0 \forall i, j, k, r, p \end{cases} \tag{11}$$

Applying the properties of the expected value method, (11) becomes

$$\begin{cases} \text{Min } Z_n = \dfrac{N_n(x)}{D_n(x)} = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P} E(\tilde{C}_{ijkrp})x_{ijkrp}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P} E(\tilde{D}_{ijkrp})x_{ijkrp}} \\ \text{subject to} \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} - E(\tilde{a}_{ip}) \le 0, i = 1,2,...I, p = 1,2,..P \\ E(\tilde{b}_{jp})\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} \le 0, j = 1,2,...J, p = 1,2,..P \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{p=1}^{P} x_{ijkrp} - E(\tilde{e}_k) \le 0, k = 1,2,...K \\ x_{ijkrp} \ge 0 \forall i, j, k, r, p \end{cases} \tag{12}$$

6. Chance Constrained Method for UMOMI4DFTP

In this section, we have presented the equivalent deterministic model for UMOMI4DFTP using chance constrained method.

Suppose that $\tilde{C}_{ijkrg}, \tilde{D}_{ijkg}, \tilde{a}_{ip}, \tilde{b}_{jp}$ and $\tilde{e}_k$ are independent uncertain variables with regular uncertainty distribution $\chi_{ijkrp}, \phi_{ijkrp}, \psi_{ip}, \theta_{jp}, \lambda_k$ respectively. (10)'s equivalent deterministic model using the chance constrained method is given as below.

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$$\begin{cases} \text{Min } Z_n^* = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P}\chi_{ijkrp}^{-1}(\alpha_n)x_{ijkrp}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P}\phi_{ijkrp}^{-1}(\gamma_n)x_{ijkrp}} \\ \text{subject to} \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}x_{ijkrp} \le \psi_{ip}^{-1}(1-\beta_1), i=1,2,...I, p=1,2,..P \\ \sum_{i=1}\sum_{k=1}\sum_{r=1}x_{ijkrp} \ge \theta_{jp}^{-1}(\beta_2), j=1,2,...J, p=1,2,..P \\ \sum_{i=1}\sum_{j=1}\sum_{r=1}\sum_{p=1}x_{ijkrp} \le \lambda_k^{-1}(1-\beta_3), k=1,2,...K \\ x_{ijkrp} \ge 0 \forall i,j,k,r,p \end{cases} \tag{13}$$

Where $\alpha_n, \gamma_n, \beta_1, \beta_2$ *and* β_3 are predetermined confidence level and $\alpha_n, \gamma_n, \beta_1, \beta_2$ and $\beta_3 \in (0,1), \forall n.$

According to the properties of chance constraint method,(15) becomes as follows

$$\begin{cases} \text{Min } Z_n^* = \dfrac{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P}e_{ijkrp} + \dfrac{\sigma_{ijkrp}}{\pi} * \sqrt{3}\ln\dfrac{\alpha_n}{1-\alpha_n}}{\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{p=1}^{P}e_{ijkrp}^1 + \dfrac{\sigma_{ijkrp}^1}{\pi} * \sqrt{3}\ln\dfrac{\gamma_n}{1-\gamma_n}} \\ \text{subject to} \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R}x_{ijkrp} \le e_{ip} + \dfrac{\sigma_{ip}\sqrt{3}}{\pi}\ln\dfrac{\alpha_{ip}}{1-\alpha_{ip}}, i=1,2,...I, p=1,2,..P \\ \sum_{i=1}\sum_{k=1}\sum_{r=1}x_{ijkrp} \ge e_{jp} + \dfrac{\sigma_{jp}\sqrt{3}}{\pi}\ln\dfrac{\beta_{ip}}{1-\beta_{ip}}, j=1,2,...J, p=1,2,..P \\ \sum_{i=1}\sum_{j=1}\sum_{r=1}\sum_{p=1}x_{ijkrp} \le e_k + \dfrac{\sigma_k\sqrt{3}}{\pi}\ln\dfrac{1-\beta_k}{\beta_k}, k=1,2,...K \\ x_{ijkrp} \ge 0 \forall i,j,k,r,p \end{cases} \tag{14}$$

Definition 10: A point $x^0 \in X$ is said to be an efficient solution of UMOMI4DFTP iff there does not exist another $x \in X$ such that $Z_n(x) \le Z_n(x^0)$, and $Z_n(x) < Z_n(x^0)$ for at least one n.

Theorem4:

$$Z^0 = \frac{N(x^0)}{D(x^0)} = \max\left\{\frac{N(x)}{D(x)} : x \in X\right\} \text{ iff } f(Z^0) = f(z^0, x^0) = \max\left\{N(x) - Z^0 D(x) : x \in X\right\}$$

According to Guzel, equation (12) becomes

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$$\begin{cases} \min\left\{\sum_{n=1}^{N}(N_n(x)-Z_n^0 D_n(x))\right\} \\ \text{subject} \quad \text{to} \\ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} - E(\tilde{a}_{ip}) \le 0, i=1,2,...I, \; p=1,2,..P \\ E(\tilde{b}_{jp}) - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} x_{ijkrp} \le 0, \; j=1,2,...J, \; p=1,2,..P \\ \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{p=1}^{P} x_{ijkrp} - E(\tilde{e}_k) \le 0, k=1,2,...K \\ x_{ijkrp} \ge 0 \forall i, j, k, r, p \end{cases} \tag{15}$$

Where $Z_n^0 = \dfrac{N_n(x^0)}{D_n(x^0)} = \min\left\{\dfrac{N_n(x)}{D_n(x)} : x \in X, n=1,2,...N\right\}$

7. Solving Algorithm for UMOMI4DFTP

In this section, we summarize the procedure to solve uncertain multi-objective multi-item four-dimensional transportation problem as in the following steps.

Step1: Formulate the UMOMI4DFTP.

Step 2: Formulate the equivalent deterministic MOMI4DFTP for the proposed UMOMI4DFTP by using expected value and chance constrained method as of (12) and (14).

Step 3: Find individual minimum value of $\boldsymbol{z_n}$, (n=1,2…N) by solving each objective function subject to the given constraints.

Step 4: On the basis of the method proposed by NuranGuzel[8] (by theorem4), equivalent linear programming problem for the proposed model is obtained as of (15).

Step 5: Solve the linear programming model obtained in step 4 using generalized reduced gradient technique (LINGO-18.0 Suite Solver) to obtain the optimal solution which is the efficient solution for UMOMI4DFTP problem.

8. Numerical Example

To showcase the effectiveness of the method discussed above, we have considered a problem. The problem is considered in which all the parameters are of the uncertain normal variables.

In the problem we have taken, we have considered four origins four destinations along with two possible routes, mode of transports and types of products each. The main aim of this problem is to minimize the ratio of the actual transportation cost and standard transportation cost, to minimize the ratio of actual transportation time and standard transportation time and to minimize the ratio of actual damage charges and standard damage charges.

The data for availabilities of goods in the origins has been presented under Table 1.

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Table 1: Availabilities in origins

i	p	$\tilde{a}_{ip}$
1	1	N(173,0.5)
	2	N(100,1)
2	1	N(260,0.2)
	2	N(225,0.8)
3	1	N(100,0.7)
	2	N(150,2)
4	1	N(100,1)
	2	N(350,2)

Table 2 contains the values of demands in the destinations**.**

Table 2: Demands in the destination

j	p	$\tilde{b}_{jp}$
1	1	N(50,0.5)
	2	N(200,0.4)
2	1	N(90,0.7)
	2	N(25,0.8)
3	1	N(50,1)
	2	N(100,2)
4	1	N(200,1.5)
	2	N(80,2)

Capacities of different conveyances have been given under table 3.

Table 3: Capacity of transports

k	r	$\tilde{e}_{kr}$
1	1	N(180,1)
	2	N(280,2)
2	1	N(180,1)
	2	N(280,0.5)

Table 4 contains the ratios of actual unit transportation cost and standard unit transportation cost,ratios of actual transportation time and standard transportation time and the ratios of the actual damage charge per unit and the standard damage charge per unit.

Table 4: Ratios of actual and standard unit transportation cost, actual and standard transportation time and actual and the standard damage charge per unit

i	j	k	r	Ratio of actual unit transportation cost and standard unit transportation cost		Ratio of actual transportation time and standard transportation time		Ratio of the actual damage charge per unit to the standard damage charge per unit	
				$\frac{\tilde{C}_{ijkr1}}{\tilde{D}_{ijkr1}}$	$\frac{\tilde{C}_{ijkr2}}{\tilde{D}_{ijkr2}}$	$\frac{\tilde{T}^A_{ijkr1}}{\tilde{T}^S_{ijkr1}}$	$\frac{\tilde{T}^A_{ijkr2}}{\tilde{T}^S_{ijkr2}}$	$\frac{\tilde{D}^A_{ijkr1}}{\tilde{D}^S_{ijkr1}}$	$\frac{\tilde{D}^A_{ijkr2}}{\tilde{D}^S_{ijkr2}}$
1	1	1	1	$\frac{N(8,0.1)}{N(18,1)}$	$\frac{N(12,0.3)}{N(22,0.2)}$	$\frac{N(8,0.1)}{N(18,1)}$	$\frac{N(32,0.3)}{N(22,0.2)}$	$\frac{N(48,0.1)}{N(38,1)}$	$\frac{N(52,0.3)}{N(32,0.2)}$
			2	$\frac{N(11,0.5)}{N(18,0.5)}$	$\frac{N(18,0.7)}{N(28,0.9)}$	$\frac{N(18,0.5)}{N(8,0.5)}$	$\frac{N(12,0.7)}{N(10,0.9)}$	$\frac{N(38,0.5)}{N(18,0.5)}$	$\frac{N(32,0.7)}{N(12,0.9)}$
		2	1	$\frac{N(18,0.9)}{N(19,0.6)}$	$\frac{N(19,1.1)}{N(29,0.5)}$	$\frac{N(29,0.9)}{N(19,0.6)}$	$\frac{N(39,1.1)}{N(39,0.5)}$	$\frac{N(39,0.9)}{N(39,0.6)}$	$\frac{N(29,1.1)}{N(29,0.5)}$
			2	$\frac{N(15,1.3)}{N(19,1.3)}$	$\frac{N(12,1.5)}{N(19,2)}$	$\frac{N(49,1.3)}{N(19,1.3)}$	$\frac{N(39,1.5)}{N(35,2)}$	$\frac{N(29,1.3)}{N(19,1.3)}$	$\frac{N(19,1.5)}{N(39,2)}$
	2	1	1	$\frac{N(14,1.7)}{N(17,0.2)}$	$\frac{N(24,1.9)}{N(29,0.4)}$	$\frac{N(27,1.7)}{N(22,0.2)}$	$\frac{N(49,1.9)}{N(29,0.4)}$	$\frac{N(37,1.7)}{N(7,0.2)}$	$\frac{N(49,1.9)}{N(29,0.4)}$
			2	$\frac{N(22,0.2)}{N(29,0.6)}$	$\frac{N(32,0.4)}{N(29,0.8)}$	$\frac{N(29,0.2)}{N(19,0.6)}$	$\frac{N(29,0.4)}{N(9,0.8)}$	$\frac{N(59,0.2)}{N(49,0.6)}$	$\frac{N(39,0.4)}{N(19,0.8)}$
		2	1	$\frac{N(29,0.6)}{N(19,1)}$	$\frac{N(25,0.8)}{N(24,1.2)}$	$\frac{N(9,0.6)}{N(9,1)}$	$\frac{N(20,0.8)}{N(10,1.2)}$	$\frac{N(19,0.6)}{N(29,1)}$	$\frac{N(20,0.8)}{N(20,1.2)}$
			2	$\frac{N(23,1)}{N(29,1.4)}$	$\frac{N(21,1.2)}{N(24,1.6)}$	$\frac{N(13,1)}{N(23,1.4)}$	$\frac{N(14,1.2)}{N(15,1.6)}$	$\frac{N(23,1)}{N(13,1.4)}$	$\frac{N(24,1.2)}{N(14,1.6)}$
	3	1	1	$\frac{N(39,1.4)}{N(29,1.8)}$	$\frac{N(18,1.6)}{N(28,2)}$	$\frac{N(25,1.4)}{N(27,1.8)}$	$\frac{N(23,1.6)}{N(33,2)}$	$\frac{N(15,1.4)}{N(25,1.8)}$	$\frac{N(13,1.6)}{N(13,2)}$
			2	$\frac{N(19,1.8)}{N(29,0.1)}$	$\frac{N(19,2)}{N(29,0.3)}$	$\frac{N(26,1.8)}{N(16,0.1)}$	$\frac{N(26,2)}{N(16,0.3)}$	$\frac{N(26,1.8)}{N(16,0.1)}$	$\frac{N(36,2)}{N(26,0.3)}$
		2	1	$\frac{N(11,0.3)}{N(21,0.5)}$	$\frac{N(14,0.6)}{N(24,0.7)}$	$\frac{N(31,0.3)}{N(11,0.5)}$	$\frac{N(14,0.6)}{N(15,0.7)}$	$\frac{N(41,0.3)}{N(31,0.5)}$	$\frac{N(34,0.6)}{N(44,0.7)}$

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			2	$\frac{N(14,0.9)}{N(24,0.9)}$	$\frac{N(34,1.2)}{N(24,1.1)}$	$\frac{N(14,0.9)}{N(17,0.9)}$	$\frac{N(34,1.2)}{N(24,1.1)}$	$\frac{N(24,0.9)}{N(14,0.9)}$	$\frac{N(44,1.2)}{N(54,1.1)}$
	4	1	1	$\frac{N(18,1.5)}{N(28,1.3)}$	$\frac{N(18,1.8)}{N(25,1.5)}$	$\frac{N(28,1.5)}{N(18,1.3)}$	$\frac{N(25,1.8)}{N(15,1.5)}$	$\frac{N(18,1.5)}{N(28,1.3)}$	$\frac{N(15,1.8)}{N(25,1.5)}$
			2	$\frac{N(25,0.4)}{N(28,1.7)}$	$\frac{N(20,0.8)}{N(23,1.9)}$	$\frac{N(28,0.4)}{N(18,1.7)}$	$\frac{N(23,0.8)}{N(13,1.9)}$	$\frac{N(18,0.4)}{N(28,1.7)}$	$\frac{N(13,0.8)}{N(23,1.9)}$
		2	1	$\frac{N(22,1.2)}{N(28,0.2)}$	$\frac{N(35,1.6)}{N(25,0.4)}$	$\frac{N(28,1.2)}{N(18,0.2)}$	$\frac{N(25,1.6)}{N(35,0.4)}$	$\frac{N(38,1.2)}{N(18,0.2)}$	$\frac{N(15,1.6)}{N(25,0.4)}$
			2	$\frac{N(30,2)}{N(28,0.6)}$	$\frac{N(18,0.5)}{N(28,0.8)}$	$\frac{N(28,2)}{N(18,0.6)}$	$\frac{N(28,0.5)}{N(18,0.8)}$	$\frac{N(18,2)}{N(18,0.6)}$	$\frac{N(18,0.5)}{N(8,0.8)}$
2	1	1	1	$\frac{N(37,1)}{N(27,1)}$	$\frac{N(15,1.5)}{N(25,1.2)}$	$\frac{N(27,1)}{N(17,1)}$	$\frac{N(25,1.5)}{N(15,1.2)}$	$\frac{N(17,1)}{N(7,1)}$	$\frac{N(35,1.5)}{N(25,1.2)}$
			2	$\frac{N(28,2)}{N(25,1.4)}$	$\frac{N(28,0.6)}{N(25,1.6)}$	$\frac{N(25,2)}{N(18,1.4)}$	$\frac{N(25,0.6)}{N(22,1.6)}$	$\frac{N(45,2)}{N(35,1.4)}$	$\frac{N(55,0.6)}{N(25,1.6)}$
		2	1	$\frac{N(17,1.2)}{N(16,1.8)}$	$\frac{N(20,1.8)}{N(21,2)}$	$\frac{N(6,1.2)}{N(16,1.8)}$	$\frac{N(11,1.8)}{N(31,2)}$	$\frac{N(16,1.2)}{N(26,1.8)}$	$\frac{N(31,1.8)}{N(11,2)}$
			2	$\frac{N(17,0.7)}{N(16,0.3)}$	$\frac{N(18,1.4)}{N(21,0.5)}$	$\frac{N(26,0.7)}{N(16,0.3)}$	$\frac{N(21,1.4)}{N(11,0.5)}$	$\frac{N(16,0.7)}{N(26,0.3)}$	$\frac{N(21,1.4)}{N(11,0.5)}$
	2	1	1	$\frac{N(18,0.8)}{N(19,0.7)}$	$\frac{N(20,1.6)}{N(21,0.9)}$	$\frac{N(29,0.8)}{N(19,0.7)}$	$\frac{N(17,1.6)}{N(13,0.9)}$	$\frac{N(39,0.8)}{N(29,0.7)}$	$\frac{N(27,1.6)}{N(17,0.9)}$
			2	$\frac{N(17,0.9)}{N(19,1.1)}$	$\frac{N(16,1.8)}{N(19,1.3)}$	$\frac{N(14,0.9)}{N(24,1.1)}$	$\frac{N(15,1.8)}{N(18,1.3)}$	$\frac{N(24,0.9)}{N(14,1.1)}$	$\frac{N(35,1.8)}{N(25,1.3)}$
		2	1	$\frac{N(15,0.1)}{N(16,1.5)}$	$\frac{N(20,0.2)}{N(21,1.7)}$	$\frac{N(18,0.1)}{N(8,1.5)}$	$\frac{N(11,0.2)}{N(21,1.7)}$	$\frac{N(8,0.1)}{N(8,1.5)}$	$\frac{N(21,0.2)}{N(11,1.7)}$
			2	$\frac{N(18,0.3)}{N(21,1.9)}$	$\frac{N(20,0.4)}{N(21,0.2)}$	$\frac{N(32,0.3)}{N(12,1.9)}$	$\frac{N(21,0.4)}{N(21,0.2)}$	$\frac{N(22,0.3)}{N(12,1.9)}$	$\frac{N(31,0.4)}{N(11,0.2)}$
	3	1	1	$\frac{N(23,0.5)}{N(28,0.4)}$	$\frac{N(22,0.6)}{N(21,0.6)}$	$\frac{N(28,0.5)}{N(18,0.4)}$	$\frac{N(21,0.6)}{N(21,0.6)}$	$\frac{N(18,0.5)}{N(28,0.4)}$	$\frac{N(11,0.6)}{N(21,0.6)}$

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			2	$\frac{N(24,0.7)}{N(28,0.8)}$	$\frac{N(21,0.8)}{N(21,1)}$	$\frac{N(18,0.7)}{N(15,0.8)}$	$\frac{N(11,0.8)}{N(11,1)}$	$\frac{N(28,0.7)}{N(38,0.8)}$	$\frac{N(21,0.8)}{N(11,1)}$
		2	1	$\frac{N(25,0.9)}{N(27,1.2)}$	$\frac{N(25,1)}{N(21,1.4)}$	$\frac{N(17,0.9)}{N(14,1.2)}$	$\frac{N(25,1)}{N(22,1.4)}$	$\frac{N(27,0.9)}{N(17,1.2)}$	$\frac{N(22,1)}{N(12,1.4)}$
			2	$\frac{N(21,1.1)}{N(27,1.6)}$	$\frac{N(23,1.2)}{N(27,1.8)}$	$\frac{N(22,1.1)}{N(23,1.6)}$	$\frac{N(22,1.2)}{N(21,1.8)}$	$\frac{N(21,1.1)}{N(20,1.6)}$	$\frac{N(23,1.2)}{N(21,1.8)}$
	4	1	1	$\frac{N(26,1.3)}{N(21,2)}$	$\frac{N(24,1.4)}{N(21,0.1)}$	$\frac{N(20,1.3)}{N(22,2)}$	$\frac{N(21,1.4)}{N(22,0.1)}$	$\frac{N(22,1.3)}{N(21,2)}$	$\frac{N(11,1.4)}{N(21,0.1)}$
			2	$\frac{N(16,1.5)}{N(18,0.3)}$	$\frac{N(25,1.6)}{N(27,0.5)}$	$\frac{N(18,1.5)}{N(17,0.3)}$	$\frac{N(27,1.6)}{N(22,0.5)}$	$\frac{N(18,1.5)}{N(17,0.3)}$	$\frac{N(57,1.6)}{N(37,0.5)}$
		2	1	$\frac{N(12,1.8)}{N(13,0.7)}$	$\frac{N(23,2)}{N(27,0.9)}$	$\frac{N(13,1.8)}{N(18,0.7)}$	$\frac{N(27,2)}{N(25,0.9)}$	$\frac{N(63,1.8)}{N(33,0.7)}$	$\frac{N(17,2)}{N(17,0.9)}$
			2	$\frac{N(12,0.2)}{N(18,1.1)}$	$\frac{N(25,0.4)}{N(27,1.3)}$	$\frac{N(18,0.2)}{N(14,1.1)}$	$\frac{N(17,0.4)}{N(12,1.3)}$	$\frac{N(12,0.2)}{N(15,1.1)}$	$\frac{N(37,0.4)}{N(27,1.3)}$
3	1	1	1	$\frac{N(35,0.6)}{N(33,1.5)}$	$\frac{N(30,0.8)}{N(27,1.7)}$	$\frac{N(23,0.6)}{N(23,1.5)}$	$\frac{N(27,0.8)}{N(22,1.7)}$	$\frac{N(33,0.6)}{N(13,1.5)}$	$\frac{N(22,0.8)}{N(20,1.7)}$
			2	$\frac{N(15,1)}{N(18,1.9)}$	$\frac{N(22,1.2)}{N(27,2)}$	$\frac{N(12,1)}{N(11,1.9)}$	$\frac{N(27,1.2)}{N(22,2)}$	$\frac{N(13,1)}{N(12,1.9)}$	$\frac{N(22,1.2)}{N(26,2)}$
		2	1	$\frac{N(16,1.4)}{N(18,2)}$	$\frac{N(14,1.6)}{N(18,1.8)}$	$\frac{N(18,1.4)}{N(13,2)}$	$\frac{N(19,1.6)}{N(14,1.8)}$	$\frac{N(38,1.4)}{N(34,2)}$	$\frac{N(19,1.6)}{N(17,1.8)}$
			2	$\frac{N(11,1.8)}{N(13,1.6)}$	$\frac{N(29,2)}{N(27,1.4)}$	$\frac{N(12,1.8)}{N(11,1.6)}$	$\frac{N(17,2)}{N(27,1.4)}$	$\frac{N(42,1.8)}{N(32,1.6)}$	$\frac{N(27,2)}{N(25,1.4)}$
	2	1	1	$\frac{N(24,0.3)}{N(27,1.2)}$	$\frac{N(25,0.6)}{N(27,1)}$	$\frac{N(27,0.3)}{N(27,1.2)}$	$\frac{N(27,0.6)}{N(37,1)}$	$\frac{N(22,0.3)}{N(21,1.2)}$	$\frac{N(25,0.6)}{N(22,1)}$
			2	$\frac{N(12,0.9)}{N(13,0.8)}$	$\frac{N(22,1.2)}{N(23,0.6)}$	$\frac{N(23,0.9)}{N(21,0.8)}$	$\frac{N(23,1.2)}{N(24,0.6)}$	$\frac{N(21,0.9)}{N(20,0.8)}$	$\frac{N(21,1.2)}{N(20,0.6)}$
		2	1	$\frac{N(21,1.5)}{N(23,0.4)}$	$\frac{N(22,1.8)}{N(21,0.2)}$	$\frac{N(23,1.5)}{N(20,0.4)}$	$\frac{N(21,1.8)}{N(20,0.2)}$	$\frac{N(20,1.5)}{N(21,0.4)}$	$\frac{N(20,1.8)}{N(24,0.2)}$

			2	$\frac{N(18,0.4)}{N(14,0.2)}$	$\frac{N(31,0.8)}{N(33,0.4)}$	$\frac{N(14,0.4)}{N(12,0.2)}$	$\frac{N(33,0.8)}{N(13,0.4)}$	$\frac{N(19,0.4)}{N(16,0.2)}$	$\frac{N(23,0.8)}{N(22,0.4)}$
	3	1	1	$\frac{N(12,1.2)}{N(14,0.6)}$	$\frac{N(14,1.6)}{N(14,0.8)}$	$\frac{N(18,1.2)}{N(16,0.6)}$	$\frac{N(18,1.6)}{N(17,0.8)}$	$\frac{N(20,1.2)}{N(21,0.6)}$	$\frac{N(28,1.6)}{N(23,0.8)}$
			2	$\frac{N(18,2)}{N(18,1)}$	$\frac{N(21,0.5)}{N(23,1.2)}$	$\frac{N(18,2)}{N(13,1)}$	$\frac{N(23,0.5)}{N(21,1.2)}$	$\frac{N(16,2)}{N(17,1)}$	$\frac{N(21,0.5)}{N(20,1.2)}$
		2	1	$\frac{N(22,1)}{N(23,1.4)}$	$\frac{N(21,1.5)}{N(23,1.6)}$	$\frac{N(23,1)}{N(11,1.4)}$	$\frac{N(23,1.5)}{N(21,1.6)}$	$\frac{N(20,1)}{N(27,1.4)}$	$\frac{N(21,1.5)}{N(20,1.6)}$
			2	$\frac{N(25,1.5)}{N(27,1.8)}$	$\frac{N(23,0.6)}{N(27,2)}$	$\frac{N(27,2)}{N(27,1.8)}$	$\frac{N(23,0.6)}{N(23,2)}$	$\frac{N(22,2)}{N(21,1.8)}$	$\frac{N(20,0.6)}{N(22,2)}$
	4	1	1	$\frac{N(24,1.2)}{N(27,0.3)}$	$\frac{N(35,1.8)}{N(27,0.6)}$	$\frac{N(27,1.2)}{N(17,0.3)}$	$\frac{N(27,1.8)}{N(17,0.6)}$	$\frac{N(25,1.2)}{N(22,0.3)}$	$\frac{N(27,1.8)}{N(25,0.6)}$
			2	$\frac{N(28,0.7)}{N(28,0.9)}$	$\frac{N(27,1.4)}{N(27,1.2)}$	$\frac{N(28,0.7)}{N(18,0.9)}$	$\frac{N(27,1.4)}{N(22,1.2)}$	$\frac{N(28,0.7)}{N(25,0.9)}$	$\frac{N(22,1.4)}{N(21,1.2)}$
		2	1	$\frac{N(28,0.8)}{N(28,1.5)}$	$\frac{N(28,1.6)}{N(28,1.8)}$	$\frac{N(28,0.8)}{N(23,1.5)}$	$\frac{N(28,1.6)}{N(27,1.8)}$	$\frac{N(26,0.8)}{N(22,1.5)}$	$\frac{N(24,1.6)}{N(22,1.8)}$
			2	$\frac{N(27,0.9)}{N(27,0.4)}$	$\frac{N(27,1.8)}{N(27,0.8)}$	$\frac{N(22,0.9)}{N(20,0.4)}$	$\frac{N(27,1.8)}{N(22,0.8)}$	$\frac{N(20,0.9)}{N(16,0.4)}$	$\frac{N(37,1.8)}{N(27,0.8)}$
4	1	1	1	$\frac{N(27,0.1)}{N(27,1.2)}$	$\frac{N(27,0.3)}{N(27,1.6)}$	$\frac{N(27,0.1)}{N(27,1.2)}$	$\frac{N(27,0.3)}{N(27,1.6)}$	$\frac{N(27,0.1)}{N(27,1.2)}$	$\frac{N(27,0.3)}{N(22,1.6)}$
			2	$\frac{N(23,0.5)}{N(23,2)}$	$\frac{N(21,0.7)}{N(21,0.5)}$	$\frac{N(23,0.5)}{N(21,2)}$	$\frac{N(20,0.7)}{N(22,0.5)}$	$\frac{N(23,0.5)}{N(21,2)}$	$\frac{N(24,0.7)}{N(22,0.5)}$
		2	1	$\frac{N(23,0.9)}{N(23,1)}$	$\frac{N(21,1.1)}{N(21,1.5)}$	$\frac{N(22,0.9)}{N(21,1)}$	$\frac{N(21,1.1)}{N(20,1.5)}$	$\frac{N(21,0.9)}{N(20,1)}$	$\frac{N(20,1.1)}{N(22,1.5)}$
			2	$\frac{N(23,1.3)}{N(23,2)}$	$\frac{N(21,1.5)}{N(21,0.6)}$	$\frac{N(23,1.3)}{N(21,2)}$	$\frac{N(21,1.5)}{N(29,0.6)}$	$\frac{N(32,1.3)}{N(22,2)}$	$\frac{N(26,1.5)}{N(25,0.6)}$
	2	1	1	$\frac{N(23,1.7)}{N(23,1.2)}$	$\frac{N(23,1.9)}{N(23,1.8)}$	$\frac{N(23,1.7)}{N(20,1.2)}$	$\frac{N(23,1.9)}{N(23,1.8)}$	$\frac{N(23,1.7)}{N(21,1.2)}$	$\frac{N(26,1.9)}{N(22,1.8)}$

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			2	$\frac{N(23,0.2)}{N(23,0.7)}$	$\frac{N(21,0.4)}{N(21,1.4)}$	$\frac{N(23,0.2)}{N(23,0.7)}$	$\frac{N(21,0.4)}{N(16,1.4)}$	$\frac{N(22,0.2)}{N(20,0.7)}$	$\frac{N(27,0.4)}{N(27,1.4)}$
		2	1	$\frac{N(21,0.6)}{N(21,0.8)}$	$\frac{N(21,0.8)}{N(21,1.6)}$	$\frac{N(21,0.6)}{N(21,0.8)}$	$\frac{N(21,0.8)}{N(21,1.6)}$	$\frac{N(27,0.6)}{N(23,0.8)}$	$\frac{N(24,0.8)}{N(24,1.6)}$
			2	$\frac{N(11,1)}{N(11,0.9)}$	$\frac{N(28,1.2)}{N(28,1.8)}$	$\frac{N(11,1)}{N(21,0.9)}$	$\frac{N(28,1.2)}{N(23,1.8)}$	$\frac{N(21,1)}{N(21,0.9)}$	$\frac{N(29,1.2)}{N(27,1.8)}$
	3	1	1	$\frac{N(11,1.4)}{N(11,0.1)}$	$\frac{N(28,1.6)}{N(28,0.3)}$	$\frac{N(11,1.4)}{N(11,0.1)}$	$\frac{N(28,1.6)}{N(28,0.3)}$	$\frac{N(51,1.4)}{N(41,0.1)}$	$\frac{N(28,1.6)}{N(28,0.3)}$
			2	$\frac{N(19,1.8)}{N(19,0.5)}$	$\frac{N(27,2)}{N(27,0.7)}$	$\frac{N(19,1.8)}{N(19,0.5)}$	$\frac{N(27,2)}{N(27,0.7)}$	$\frac{N(49,1.8)}{N(39,0.5)}$	$\frac{N(35,2)}{N(25,0.7)}$
		2	1	$\frac{N(19,1.8)}{N(19,0.9)}$	$\frac{N(19,1.6)}{N(19,1.1)}$	$\frac{N(19,1.8)}{N(19,0.9)}$	$\frac{N(19,1.6)}{N(19,1.1)}$	$\frac{N(29,1.8)}{N(29,0.9)}$	$\frac{N(39,1.6)}{N(37,1.1)}$
			2	$\frac{N(11,1.4)}{N(11,1.3)}$	$\frac{N(28,1.2)}{N(28,1.5)}$	$\frac{N(11,1.4)}{N(11,1.3)}$	$\frac{N(28,1.2)}{N(28,1.5)}$	$\frac{N(21,1.4)}{N(21,1.3)}$	$\frac{N(24,1.2)}{N(22,1.5)}$
	4	1	1	$\frac{N(28,1)}{N(28,1.7)}$	$\frac{N(28,0.8)}{N(28,1.9)}$	$\frac{N(24,1)}{N(34,1.7)}$	$\frac{N(23,0.8)}{N(23,1.9)}$	$\frac{N(24,1)}{N(22,1.7)}$	$\frac{N(21,0.8)}{N(21,1.9)}$
			2	$\frac{N(18,0.6)}{N(18,0.2)}$	$\frac{N(25,0.4)}{N(25,0.4)}$	$\frac{N(16,0.6)}{N(16,0.2)}$	$\frac{N(25,0.4)}{N(25,0.4)}$	$\frac{N(46,0.6)}{N(46,0.2)}$	$\frac{N(35,0.4)}{N(35,0.4)}$
		2	1	$\frac{N(18,0.2)}{N(18,0.6)}$	$\frac{N(27,0.1)}{N(27,0.8)}$	$\frac{N(18,0.2)}{N(19,0.6)}$	$\frac{N(27,0.1)}{N(27,0.8)}$	$\frac{N(28,0.2)}{N(28,0.6)}$	$\frac{N(27,0.1)}{N(27,0.8)}$
			2	$\frac{N(28,0.3)}{N(28,1)}$	$\frac{N(25,0.5)}{N(25,1.2)}$	$\frac{N(28,0.3)}{N(28,1)}$	$\frac{N(35,0.5)}{N(32,1.2)}$	$\frac{N(18,0.3)}{N(28,1)}$	$\frac{N(55,0.5)}{N(35,1.2)}$

To apply the following algorithm, we may follow the below steps sequentially
Step 1: For the data considered above, formulate the UMOMI4DTP as of (10).
Step 2: By applying the expected value method, obtain the corresponding optimization problem of the UMOMI4DFTP as of (12).

Step 3: Solve the given three objectives individually to obtain the optimum values of each objective. The optimum values of each objectives which have been obtained has been presented in the table (5).

Table 5: Optimum values of the objectives

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Objectives	Objective values	Variable Values
Min Z_1	0.6689	$x_{11121}=50, x_{13211}=123, x_{13212}=28, x_{14222}=71$ $x_{21112}=180, x_{22122}=25, x_{24212}=8.5, x_{24221}=200$ $x_{31212}=20, x_{32121}=90, x_{33122}=63, x_{33222}=8.5$
Min Z_2	0.6534	$x_{11111}=50, x_{12221}=73, x_{12222}=25, x_{13112}=55$ $x_{13221}=50, x_{21212}=150, x_{22121}=30, x_{23122}=45$ $x_{24121}=125, x_{31222}=117, x_{42221}=25, x_{44111}=75, x_{44122}=80$
Min Z_3	0.6306	$x_{11222}=100, x_{12211}=73, x_{14121}=100, x_{21211}=15$ $x_{21221}=80, x_{22211}=17, x_{23112}=100, x_{24112}=80$ $x_{31122}=100, x_{32212}=25, x_{33211}=50, x_{44221}=100$

Step 4: According to the theorem (4) developed by Nuran Guzel, obtain the corresponding linear programming problem to the proposed UMOMI4DFTP as of (15).

Step 5: Solve the above said problem obtained in step 4 by using the generalized reduced gradient technique (LINGO-18.0 suit solver) to obtain the optimal solution. The solution obtained has been given in the table 6.

Table 6: Efficient solution of the objectives from expected value method

Variable Values	Objective values
$x_{12211}=25, x_{13111}=50, x_{14212}=80, x_{21211}=50$ $x_{23112}=100, x_{24111}=30, x_{24121}=155, x_{41222}=50$ $x_{42212}=25, x_{42221}=65, x_{44221}=15$	Min $Z_1=0.8938$ Min $Z_2=0.6917$ Min $Z_3=0.7922$

Repeat the above said steps from 1 to 5 for chance constraint method for the proposed UMOMI4DFTP to obtain the efficient solution of the model. The solution has been obtained by considering predetermined confidence level as $\alpha_1=\alpha_2=\alpha_3=\gamma_1=\gamma_2=\gamma_3=0.9, \beta_1=\beta_2=\beta_3=0.9.$ The efficient solution has been given in the table 7.

Table 7. Efficient solution of the objectives from chance constraint method

Variable Values	Objective values
$x_{13112}=77.64, x_{13212}=21.5, x_{14111}=16.05, x_{21211}=51$ $x_{21212}=53.02, x_{23112}=2.66, x_{23121}=22, x_{24112}=82.4$ $x_{24221}=185.15, x_{31122}=121.8, x_{31212}=26.9, x_{33121}=28.78, x_{42212}=26.08$	Min $Z_1=0.8309$ Min $Z_2=0.8927$ Min $Z_3=0.8833$

The usage of chance constrained method enables flexibility to the decision makers to obtain the optimal solutions as per their desired conditions. Considering diverse set of values for different parameters in the proposed model, will benefit the decision making under uncertain environment.

9. Conclusion

In this paper MOMI4DFTP problem has been investigated under uncertain environment. The model developed is very much advantageous to the decision maker, as parameters used are all in the form of uncertain variables, so that the decision maker can take realistic decisions. We have converted the UMOMI4DFTP model into corresponding crisp model by making use of expected value method and chance constraint method. The MOMI4DFTP has then been converted into single objective linear programming using Guzel's method 2013 and solved. The proposed method's validity in real world example has been tested by proving large number of choices and has been verified that the model yields optimal solution according to the decision maker's goals.

References

[1]Hitchcock FL. The distribution of a product from several sources to numerous localities, Journal of Mathematical. Physics 1941; 20: 224-230.doi: 10.1002/sapm1941201224.

[2]Lee SM., Moore LJ. Optimizing transportation problems with multiple objectives. AIIE transactions. 1973; 5(4): 333-338.doi: 10.1080/05695557308974920

[3]El-Wahed, WFA. A multi-objective transportation problem under fuzziness. Fuzzy Sets and Systems, 2001; 117(1): 27-33.doi: 10.1016/S0165-0114(98)00155-9

[4]Pramanik S. and Roy TK. Multiobjective transportation model with fuzzy parameters: priority based fuzzy goal programming approach. Journal of Transportation Systems Engineering and Information Technology. 2008; 8(3): 40-48.doi: 10.1016/S1570-6672(08)60023-9

[5]Haley BK. The solid transportation problem. Operational Research. 1962; 11: 446-448.

[6]SwarupK. Transportation technique in linear fractional functional programming. Journal of Royal Naval Scientific Service. 1966; 21(5): 256-260.

[7]Gomathi SV,JayalakshmiM. An alogorithm to solve linear fractional transportation problem. International Journal of Advanced Science and Technology.2020; 29(5): 12935-12944.

[8]Nuran Guzel. A proposalto the solution of multiobjective linear fractional programming problem. Hidnawi Publishing Corporation Abstract and Applied Analysis. Volume 2013, Article ID 435030, 4 pages.doi: 10.1155/2013/435030

[9]DinkelbachW. On nonlinear programming. Management science. 1967; 13: 492-498.doi: 10.1287/mnsc.13.7.492

[10]Zadeh LA. Fuzzy sets. Inf. Control1965; 8: 338–353.doi: 10.1016/S0019-9958(65)90241-X

[11]Gupta.A, Kumar.A and Kaur.A, Mehar's method to find exact fuzzy optimal solution of unbalancedfully fuzzy multiobjective transportation problems. Optim. Lett. 2012; 6:1737–1751.Doi: 10.1007/s11590-011-0367-2

[12]OjhaA, DasB, Mondala S and MaitiM. An entropy based solid transportation problem for generalfuzzy costs and time with fuzzy equality. Math. Comput. Model. 2009; 50:166–178.doi: 10.1016/j.mcm.2009.04.010

[13]Deepika Rani and GulatiT.R. Uncertain multi-objective multi-product solid transportation problems. Sadhana 2016; 41: 531–539.

[14]Dipak Kumar Jana et al, Novel Multi objective multi item and four dimensional transportationproblem with vehicle speed in LR type intuitionistic fuzzy

environment, Neural computing andApplications2020; 32:11937-11955.doi: 10.1007/s00521-019-04675-y

[15]Moumita D. and De PK. A note on fuzzy multi- objective linear fractional programming problem. International Journal of Computer Science and Network, 2014; 3(6): 568- 572.

[16]Dheyab AN. Finding the optimal solution for fractional linear programming problems with fuzzy numbers. Journal of Kerbala University2012; 10(3): 105- 110.

[17]Jain S. Modeling of Gauss elimination technique multi- objective fractional programming problem. South Asian Journal of Mathematics 2014; 4(3): 148- 153.

[18]Khalifa HA. and 2Al- Shabi M. Utilizing of fractional programming for multi objective multi item solid transportation problems in fuzzy environment, International Journal of Current Research,2018;10(11): 75024-75035.doi: 10.24941/ijcr.32742.11.2018

[19]Liu B. Uncertainty Theory, 2nd ed., Springer-Verlag, Berlin, (2007).

[20]Yuan Gao, Uncertain models for single facility location problems on networks, Applied MathematicalModelling 2012; 36:2592-2599.doi: 10.1016/j.apm.2011.09.042

[21]LiuB. Some Research Problems in Uncertainty Theory, Journal of Uncertain Systems, 2009; 3:3–10.

[22]Bo Zhang, Jin Peng, Uncertain programming model for uncertain optimal assignment problem. AppliedMathematical Modelling 2013; 37:6458-6468.doi: 10.1016/j.apm.2013.01.032

[23]Liu B. Uncertainty theory, Springer, Berlin, Germany, 4th edition, (2013).

[24]GaoX. Some Properties of Continuous Uncertain Measure. Internaltional Journal of Uncertainty. Fuzziness and Knowledge-Based System 2009; 17:419-426.doi: 10.1142/S0218488509005954

[25] Guo et al, A Transportation Problem with Uncertain Costs and Random Supplies, International Journal of e-Navigation and Maritime Economy, 2015; 2:1 – 11.doi: 10.1016/j.enavi.2015.06.001

[26]SeyyedMojtabaGhasemi, Mohammad Reza Safi. Linear Fractional Programming Problem in Uncertain Environments, Journal of Mathematical Analysis. 2017; 8(2): 23-33.

[27]Ali Mahmoodirad, Reza Dehghan&SadeghNiroomand. Modelling linear fractional transportation problem in belief degree—based uncertain environment. Journal of experimental & theoretical artificial intelligence,(2018).doi: 10.1080/0952813X.2018.1552318

[28]Zhou J, Fan Y, and Ke W. Multi-objective optimization in uncertain random environments. FuzzyOptimization and Decision Making 2014; 13(4):397-413.doi: 10.1007/s10700-014-9183-3

[29]Zhong S, Chen Y, Zhou J, and Liu Y. An interactive satisficing approach for multi-objective optimization with uncertain parameters. Journal of Intelligent Manufacturing. 2017; 28:535-547.doi: 10.1007/s10845-014-0998-0

[30] RevathiA.N,Mohanaselvi S. Anew solution approach to the uncertain multi-objective linear fractionalprogramming problems, AIP Conference Proceedings **2277**, 200009 (2020).doi: 10.1063/5.0025256

[31]Cui Q, Sheng Y. Uncertain programming model for solid transportation problem. An InternationalInderdisciplinaryJournal2013; 16(2(A)):1207-1214.

[32]Yang et al. Reduction methods of type-2 uncertain variables and their application to solid transportation problem. Information science, 2015; 291:204-237.doi: 10.1016/j.ins.2014.08.044

[33] LiuB. Uncertainty Theory: A Branch of Mathematics for Modelling Human Uncertainty. Springer-Verlag, Berlin, 2010.doi: 10.1007/978-3-642-13959-8_1

[34] LiuY., HaM. Expected values of function of uncertain variables. Journal of Uncertain Systems 2010; 4(3):181–186.