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Optimization of Fuzzy Inventory Model with Deterioration Using Non Linear Programming Methods

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Abstract

This paper presents an integrated inventory model with deterioration rate. The deterioration rate is playing a vital role in inventory systems. Here, we determine the minimize the total cost and maximize the optimum order time interval using non linear programming method, first we optimized the inventory model using fuzzy geometric programming method; secondly we apply the Lagrangian method for optimization. In both the methods, we use heptagonal fuzzy number for fuzzification and Pascal's Triangular Graded Mean for defuzzification, Finally, numerical examples, comparative study, sensitivity analysis, and graph are illustrated

Keywords: Geometric programming, Lagrangian method, Pascal's Triangular Graded Mean Method.

1. Introduction

An inventory management modeling exercise, demand can be represented as either deterministic or stochastic. In earlier period the uncertainties of inventory models are treated as randomness and are handled by using probability theory. Ya Yanga,c, Huihui Chib, Wei Zhoub,d, Tijun Fana, Selwyn Piramuthue[1],developed Deterioration control decision support for perishable inventory management systems .

L.A. Zadeh,[2] ,introduced fuzzy sets ,operations and applications, Chen and Wang[11] used trapezoidal fuzzy number to fuzzify the order cost, A.Mohammed Shapique[3],derived , Arithmetic Operations on heptagonal fuzzy numbers ,

Many Researches solved Integrated inventory models by Non linear programming methods. Peterson E. L, Duffin R. J,Zener C. M[6] (1967) is discovered and developed the Geometric Programming Theory and Application, Klain and Jung [7] derived single item inventory problems using geometric problems.K.Kalaiarasi,M.Sumathi,M.Sabina Begum [8],Optimized the a inventory model Using Fuzzy Geometric Programming method .
Chen's [8] function principle is proposed for arithmetic operation of fuzzy number and Lagrangian method is used for optimization. Graded mean integration is used for defuzzifying the annual integrated total cost for EPQ.

In this paper, the Mathematical model is formulated in crisp and fuzzy environment .A numerical example is encapsulating the solution in crisp and fuzzy environment. we analyse the results. Finally, we conclude this result.

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2. Methodology

2.1. Heptagonal Fuzzy Number

A heptagonal Fuzzy Number of a fuzzy set A is $\tilde{A}_h = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ its membership function is

$$\mu_{\tilde{A}_h}(x) = \begin{cases} \dfrac{(x-a_1)}{2(a_2-a_1)}, \; for a_1 \le x \le a_2; \\ \dfrac{1}{2}, \; for a_2 \le x \le a_3; \\ \dfrac{(x-a_4)}{2(a_4-a_3)}, \; for a_3 \le x \le a_4; \\ \dfrac{(a_4-x)}{2(a_5-a_4)}, \; for a_4 \le x \le a_5; \\ \dfrac{1}{2}, \; for a_5 \le x \le a_6; \\ \dfrac{(a_7-x)}{2(a_7-a_6)}, \; for a_6 \le x \le a_7; \\ 0, otherwise \end{cases}$$

2.2. Graphical Representation of Heptagonal Fuzzy Number

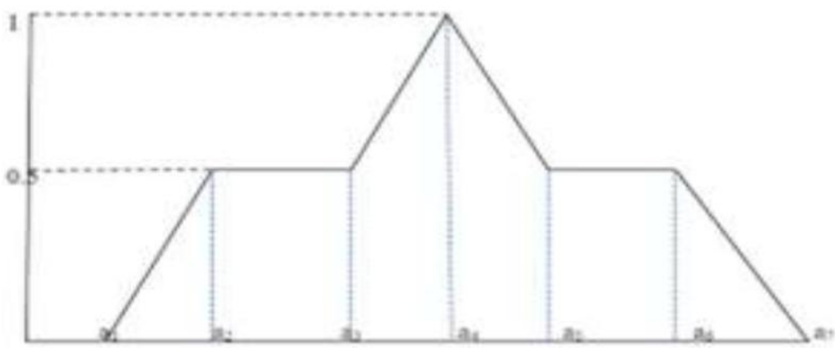

Fig 2.1 Graphical representation of Heptagonal fuzzy number

Pascal's Triangular Graded Mean for Heptagonal Fuzzy Number

Let $\tilde{A}_h = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a heptaconal fuzzy numbers, Pascal's triangles formula is

$$d(A) = \frac{a_1 + 6a_2 + 15a_3 + 20a_4 + 15a_5 + 6a_6 + a_7}{64}$$

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3. Mathematical Model

3.1. Notations

T	The Time Interval Between 2 Orders
O_C	Ordering cost
H_M	Marginal Holding Cost
D_C	Deterioration cost
TC	total cost per time unit

Table 1: Notation list

3.2. Mathematical Model

Ya Yanga,c, Huihui Chib, [1] develope the following inventory model .let us consider the proposed inventory model of the minimum total cost is

$$TC(T) = \frac{O_C}{T} + \frac{H_M D_C T}{2}$$,T>0---------------------(3.2.1)

The optimum T and TC (T) can be obtained by $\frac{\partial TC}{\partial T} = 0$

Optimal order quantity $T = \sqrt{\frac{2O_C}{H_M D_C}}$ ------------(3.2.2)

Minimum total cost $TC = \sqrt{2O_C H_M D_C}$ -------------(3.2.3)

4. Mathematical Model in crisp sence and the Geometric programming solution

let us consider the proposed inventory model of the minimum total cost is

$$TC(T) = \frac{O_C}{T} + \frac{H_M D_C T}{2}$$,T>0---------------------(4.1)

The primal problem is

Min $TC(T) = \frac{O_C}{T} + \frac{H_M D_C T}{2}$ ----------------------(4.2)

T >0

Dual problem is

Max

$$f(t) = \left(\frac{O_C}{t_1}\right)^{t_1} \left(\frac{H_M D_C}{2t_2}\right)^{t_2}$$ --------------------------------(4.3)

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Here DD = 2-1-1=0

subject to the normality and orthogonality conditions

$$\left.\begin{array}{l} t_1 + t_2 = 1 \\ -t_1 + t_2 = 0 \\ t_1, t_2 \geq 0 \end{array}\right\} \text{---(4.4)}$$

Solve the equations (4.4) we get,

$$t_1 = t_2 = \frac{1}{2} \text{--(4..5)}$$

Substitute the value of t_1, t_2 in equation (4.3), then the dual function is given by,

$$f^*(t) = \left(\frac{O_C}{\left(\frac{1}{2}\right)}\right)^{\left(\frac{1}{2}\right)} \left(\frac{H_M D_C}{2\left(\frac{1}{2}\right)}\right)^{\left(\frac{1}{2}\right)}$$

$$f^*(t) = \sqrt{2 O_C H_M D_C} \qquad \text{-----------------------(4.6)}$$

$$\frac{O_C}{T} = t_1^* f^*(t)$$

$$\frac{H_M D_C T}{2} = t_2^* f^*(t)$$

$$T^2 = \frac{2 O_C}{H_M D_C} \qquad \text{---------------------------------------(4.7)}$$

Min $TC(T) = \sqrt{2 O_C H_M D_C}$ -------------------------------(4.8)

4.1. Mathematical Model in fuzzy sence and the Geometric programming solution

Suppose $\tilde{O}_C$ *and* $\tilde{D}_C$ are taken as a fuzzy heptagonal fuzzy numbers

The primal problem is

Min $TC(T) = \dfrac{\tilde{O}_C}{T} + \dfrac{H_M \tilde{D}_C T}{2}$ ----------------------(4.1.1)

T >0

Dual problem is

Max

$$f(t) = \left(\frac{\tilde{O}_C}{t_1}\right)^{t_1} \left(\frac{H_M \tilde{D}_C}{2 t_2}\right)^{t_2} \text{--------------------------------(4.1.2)}$$

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We use Pascal's Triangular Graded Mean for Defuzzification ,

$$\tilde{O}_C = \frac{(O_{C1}+6O_{C2}+15O_{C3}+20O_{C4}+15O_{C5}+6O_{C6}+O_{C7})}{64}$$

$$\tilde{D}_C = \frac{(D_{C1}+6D_{C2}+15D_{C3}+20D_{C4}+15D_{C5}+6D_{C6}+D_{C7})}{64}$$ ----------(4.1.3)

We apply GP method we get optimum order and total cost is

$$T^2 = \frac{2(O_{C1}+6O_{C2}+15O_{C3}+20O_{C4}+15O_{C5}+6O_{C6}+O_{C7})}{H_M(D_{C1}+6D_{C2}+15D_{C3}+20D_{C4}+15D_{C5}+6D_{C6}+D_{C7})}$$ -------(4.1.4)

$$TC(T) = \left(\frac{1}{64}\right)$$

$$\left(\sqrt{2H_M(O_{C1}+6O_{C2}+15O_{C3}+20O_{C4}+15O_{C5}+6O_{C6}+O_{C7})(D_{C1}+6D_{C2}+15D_{C3}+20D_{C4}+15D_{C5}+6D_{C6}+D_{C7})}\right)$$ ---------(4.1.5)

5. Mathematical Model in Crisp sence and the Lagrange solution

let us consider the proposed inventory model of the minimum total cost is

$$TC(T) = \frac{O_C}{T} + \frac{H_M D_C T}{2}$$,T>0---------------------(5.1)

To apply Pascal's Triangular Graded Mean method to defuzzify the fuzzy total cost, and then obtain the optimal order quantity T by using new arithmetic operation

$$TC(T) = \frac{1}{64}\left(\begin{array}{l} \left(\frac{O_{C1}}{T}+\frac{H_M D_{C1} T}{2}\right)+6\left(\frac{O_{C2}}{T}+\frac{H_M D_{C2} T}{2}\right) \\ +15\left(\frac{O_{C3}}{T}+\frac{H_M D_{C3} T}{2}\right)+20\left(\frac{O_{C4}}{T}+\frac{H_M D_{C4} T}{2}\right) \\ +15\left(\frac{O_{C5}}{T}+\frac{H_M D_{C5} T}{2}\right)+6\left(\frac{O_{C6}}{T}+\frac{H_M D_{C6} T}{2}\right)+\left(\frac{O_{C7}}{T}+\frac{H_M D_{C7} T}{2}\right) \end{array}\right)$$ ---------(5.2)

Computation of T at which TC(T) is minimum, when $\frac{dTC(T)}{dT} = 0$ and where

$$\frac{d^2TC(T)}{dT^2} > 0$$

$$T^2 = \frac{2(O_{C1}+6O_{C2}+15O_{C3}+20O_{C4}+15O_{C5}+6O_{C6}+O_{C7})}{H_M(D_{C1}+6D_{C2}+15D_{C3}+20D_{C4}+15D_{C5}+6D_{C6}+D_{C7})}$$ -------(4.1.4)

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$$TC(T)=\left(\frac{1}{64}\right)$$
$$\left(\sqrt{2H_M(O_{C1}+6O_{C2}+15O_{C3}+20O_{C4}+15O_{C5}+6O_{C6}+O_{C7})(D_{C1}+6D_{C2}+15D_{C3}+20D_{C4}+15D_{C5}+6D_{C6}+D_{C7})}\right)$$

5.1. Mathematical Model in Fuzzy sence and the Lagrange solution

let us consider the proposed inventory model of the minimum total cost is

$$TC(T)=\frac{O_C}{T}+\frac{H_M D_C T}{2} \quad ,T>0 \text{---------------------}(5.1)$$

To apply Pascal's Triangular Graded Mean method to defuzzify the fuzzy total cost, and then obtain the optimal order quantity T by using new arithmetic operation

$$TC(T)=\frac{1}{64}\left(\begin{array}{l}\left(\frac{O_{C1}}{T_7}+\frac{H_M D_{C1}T_1}{2}\right)+6\left(\frac{O_{C2}}{T_6}+\frac{H_M D_{C2}T_2}{2}\right)\\ +15\left(\frac{O_{C3}}{T_5}+\frac{H_M D_{C3}T_3}{2}\right)+20\left(\frac{O_{C4}}{T_4}+\frac{H_M D_{C4}T_4}{2}\right)\\ +15\left(\frac{O_{C5}}{T_3}+\frac{H_M D_{C5}T_5}{2}\right)+6\left(\frac{O_{C6}}{T_2}+\frac{H_M D_{C6}T_6}{2}\right)+\left(\frac{O_{C7}}{T_1}+\frac{H_M D_{C7}T_7}{2}\right)\end{array}\right) \text{---------}(5.2)$$

with $0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$,

If we replace inequality conditions $0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$ into the following
$0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$
inequality $T_2-T_1\ge 0, T_3-T_2\ge 0, T_4-T_3\ge 0, T_5-T_4\ge 0, T_6-T_5\ge 0, T_7-T_6\ge 0, T_1\ge 0$

In the following stages, extension of the Lagrangian method is used to find the solutions of $T_1, T_2, T_3, T_4, T_5, T_6, T_7$ to minimize $P(T\tilde{C}(\tilde{T}))$.

Stage 1 : To find the min $P(T\tilde{C}(\tilde{T}))$,we have to find the derivative of $P(T\tilde{C}(\tilde{T}))$ with respect to $T_1, T_2, T_3, T_4, T_5, T_6, T_7$. equal to zero

Let $\frac{\delta P}{\delta T_1}=0, \frac{\delta P}{\delta T_2}=0, \frac{\delta P}{\delta T_3}=0, \frac{\delta P}{\delta T_4}=0, \frac{\delta P}{\delta T_5}=0, \frac{\delta P}{\delta T_6}=0, \frac{\delta P}{\delta T_7}=0,$ Then

$$T_1=\sqrt{\frac{2O_{C7}}{H_M D_{C1}}}, T_2=\sqrt{\frac{2O_{C6}}{H_M D_{C2}}}, T_3=\sqrt{\frac{2O_{C5}}{H_M D_{C3}}},$$
$$T_4=\sqrt{\frac{2O_{C4}}{H_M D_{C4}}}, T_5=\sqrt{\frac{2O_{C3}}{H_M D_{C5}}}, T_6=\sqrt{\frac{2O_{C2}}{H_M D_{C6}}}, T_7=\sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

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Because the above show that $0 > T_1 > T_2 > T_3 > T_4. > T_5 > T_6 > T_7$ it does not satisfy the constraint $0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$,so K = 1 and go to Stage 2.

Stage 2 : Convert the inequality constraint $T_2 - T_1 \ge 0$ into equality constraint $T_2 - T_1 = 0$. We have Lagrangian function as $L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda) = P(T\tilde{C}(\tilde{T})) - \lambda(T_2 - T_1)$. we have to find the derivative of $L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda)$ with respect to $T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda$. equal to zero.

$$T_1 = T_2 = \sqrt{\frac{2(O_{C7} + 6O_{C6})}{H_M(D_{C1} + 6D_{C2})}},$$
$$T_3 = \sqrt{\frac{2O_{C5}}{H_M D_{C3}}}, T_4 = \sqrt{\frac{2O_{C4}}{H_M D_{C4}}}, T_5 = \sqrt{\frac{2O_{C3}}{H_M D_{C5}}}, T_6 = \sqrt{\frac{2O_{C2}}{H_M D_{C6}}}, T_7 = \sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

Because the above show that $T_3 > T_4. > T_5 > T_6 > T_7$ it does not satisfy the constraint $0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$,so K = 2 and go to Stage 3.

Stage 3 : Convert the inequality constraints $T_2 - T_1 \ge 0$, $T_3 - T_2 \ge 0$, into equality constraints $T_2 - T_1 = 0$ and $T_3 - T_2 = 0$. Then the Lagrangian method is

$L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda_1,\lambda_2) = P(T\tilde{C}(\tilde{T})) - \lambda_1(T_2 - T_1) - \lambda_2(T_3 - T_2)$, we have to find the derivative of $L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda_1,\lambda_2)$ with respect to $T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1$, and λ_2 ,and equal to zero,

$$T_1 = T_2 = T_3 = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5})}{H_M(D_{C1} + 6D_{C2} + D_{C3})}},$$
$$T_4 = \sqrt{\frac{2O_{C4}}{H_M D_{C4}}}, T_5 = \sqrt{\frac{2O_{C3}}{H_M D_{C5}}}, T_6 = \sqrt{\frac{2O_{C2}}{H_M D_{C6}}}, T_7 = \sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

Because the above show that $T_4. > T_5 > T_6 > T_7$ it does not satisfy the constraint $0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$,so K = 3 and go to Stage 4.

Stage 4 : Convert the inequality constraints $T_2 - T_1 \ge 0$, $T_3 - T_2 \ge 0$, $T_4 - T_3 \ge 0$,into equality constraints $T_2 - T_1 = 0$, $T_3 - T_2 = 0$ and $T_4 - T_3 = 0$. We optimize The Lagrangean function is given by

$L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda_1,\lambda_2,\lambda_3) = P(T\tilde{C}(\tilde{T})) - \lambda_1(T_2 - T_1) - \lambda_2(T_3 - T_2) - \lambda_3(T_4 - T_3)$

we have to find the derivative of $L(T_1,T_2,T_3,T_4,T_5,\mathrm{T}_6,T_7,\lambda_1,\lambda_2,\lambda_3)$ with respect to $T_1, T_2, T_3, T_4, T_5, T_6, T_7$, $\lambda_1, \lambda_2,$ *and* λ_3,and equal to zero,

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$$T_1 = T_2 = T_3 = T_4 = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5} + 20O_{C4})}{H_M(D_{C1} + 6D_{C2} + D_{C3} + 20D_{C4})}},$$
$$T_5 = \sqrt{\frac{2O_{C3}}{H_M D_{C5}}}, T_6 = \sqrt{\frac{2O_{C2}}{H_M D_{C6}}}, T_7 = \sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

Because the above show that $T_5 > T_6 > T_7$ it does not satisfy the constraint
$0 < T_1 \le T_2 \le T_3 \le T_4 . \le T_5 \le T_6 \le T_7$,so K = 4 and go to Stage 5.

Stage 5 : Convert the inequality constraints $T_2 - T_1 \ge 0$, $T_3 - T_2 \ge 0$, $T_4 - T_3 \ge 0$,and $T_5 - T_4 \ge 0$ into equality constraints $T_2 - T_1 = 0$, $T_3 - T_2 = 0$,$T_4 - T_3 = 0$ and $T_5 - T_4 = 0$. The Lagrangean function is given by

$$L(T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = P(T\tilde{C}(\tilde{T})) - \lambda_1(T_2 - T_1) - \lambda_2(T_3 - T_2) - \lambda_3(T_4 - T_3) - \lambda_4(T_5 - T_4)$$

we have to find the derivative of $L(T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ with respect to T_1 ,T_2 , T_3 ,T_4, T_5, T_6, T_7 ,$\lambda_1, \lambda_2, \lambda_3, and \lambda_4$,and equal to zero

$$T_1 = T_2 = T_3 = T_4 = T_5 = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5} + 20O_{C4} + 15O_{C3})}{H_M(D_{C1} + 6D_{C2} + D_{C3} + 20D_{C4} + D_{C5})}},$$
$$T_6 = \sqrt{\frac{2O_{C2}}{H_M D_{C6}}}, T_7 = \sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

Because the above show that $T_6 > T_7$ it does not satisfy the constraint
$0 < T_1 \le T_2 \le T_3 \le T_4 . \le T_5 \le T_6 \le T_7$,so K = 5 and go to Stage 6.

Stage 6 : Convert the inequality constraints $T_2 - T_1 \ge 0$, $T_3 - T_2 \ge 0$, $T_4 - T_3 \ge 0$, $T_5 - T_4 \ge 0$ and $T_6 - T_5 \ge 0$ into equality constraints $T_2 - T_1 = 0$, $T_3 - T_2 = 0$,$T_4 - T_3 = 0$, $T_5 - T_4 = 0$ and $T_6 - T_5 = 0$. The Lagrangean function is given by

$$L(T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) = P(T\tilde{C}(\tilde{T})) - \lambda_1(T_2 - T_1) - \lambda_2(T_3 - T_2) - \lambda_3(T_4 - T_3) - \lambda_4(T_5 - T_4) - \lambda_5(T_6 - T_5)$$

we have to find the derivative of $L(T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ with respect to T_1 ,T_2 , T_3 ,T_4, T_5, T_6, T_7 ,$\lambda_1, \lambda_2, \lambda_3, \lambda_4, and$,and equal to zero

$$T_1 = T_2 = T_3 = T_4 = T_5 = T_6 = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5} + 20O_{C4} + 15O_{C3} + 6O_{C2})}{H_M(D_{C1} + 6D_{C2} + D_{C3} + 20D_{C4} + D_{C5} + 6D_{C6})}},$$
$$T_7 = \sqrt{\frac{2O_{C1}}{H_M D_{C7}}}$$

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Because the above show that $T_6 > T_7$ it does not satisfy the constraint

$0 < T_1 \le T_2 \le T_3 \le T_4. \le T_5 \le T_6 \le T_7$,so K = 6 and go to Stage 7.

Stage 7 : Convert the inequality constraints $T_2 - T_1 \ge 0$, $T_3 - T_2 \ge 0$, $T_4 - T_3 \ge 0$, $T_5 - T_4 \ge 0$ $T_6 - T_5 \ge 0$ and $T_7 - T_6 \ge 0$ into equality constraints $T_2 - T_1 = 0$, $T_3 - T_2 = 0$, $T_4 - T_3 = 0$, $T_5 - T_4 = 0$, $T_6 - T_5 = 0$ and $T_7 - T_6 = 0$. The Lagrangean function is given by

$$L(T_1,T_2,T_3,T_4,T_5,T_6,T_7,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6) = P(T\tilde{C}(\tilde{T})) - \lambda_1(T_2 - T_1) - \lambda_2(T_3 - T_2) - \lambda_3(T_4 - T_3) - \lambda_4(T_5 - T_4) - \lambda_5(T_6 - T_5) - \lambda_6(T_7 - T_6)$$

we have to find the derivative of $L(T_1,T_2,T_3,T_4,T_5,T_6,T_7,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6)$ with respect to $T_1, T_2, T_3, T_4, T_5, T_6, T_7, \lambda_1, \lambda_2, \lambda_3, \lambda_4, and\ \lambda_6$,and equal to zero

$$T_1 = T_2 = T_3 = T_4 = T_5 = T_6 = T_7 = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5} + 20O_{C4} + 15O_{C3} + 6O_{C2} + O_{C1})}{H_M(D_{C1} + 6D_{C2} + D_{C3} + 20D_{C4} + D_{C5} + 6D_{C6} + D_{C7})}}$$

$$T = \sqrt{\frac{2(O_{C7} + 6O_{C6} + 15O_{C5} + 20O_{C4} + 15O_{C3} + 6O_{C2} + O_{C1})}{H_M(D_{C1} + 6D_{C2} + D_{C3} + 20D_{C4} + D_{C5} + 6D_{C6} + D_{C7})}}$$

6. Numerical Example

Example:

Crisp Model:

Let us consider the following data:

O_C= 50/- Per unit , D_C = 84 unit/year, H_M = Rs.15/- Per unit,

By Minimizing The Total Cost ,Using Geometric Programming And Lagrange Method,We Obtain The Optimal Value

T =0.2817 and TC=Rs.354.96/-

Fuzzy Model:

Let us Take the fuzzy data:

$\tilde{O}_C$ = (35,40,45,50,55,60,65) , $\tilde{D}_C$ = (54,64,74,84,94,104,114),

H_M = Rs.15/- Per unit,

By Minimizing The Total Cost ,Using Geometric Programming And Lagrange Method,We Obtain The Optimal Value

T =0.2817 and TC=Rs.354.96/-

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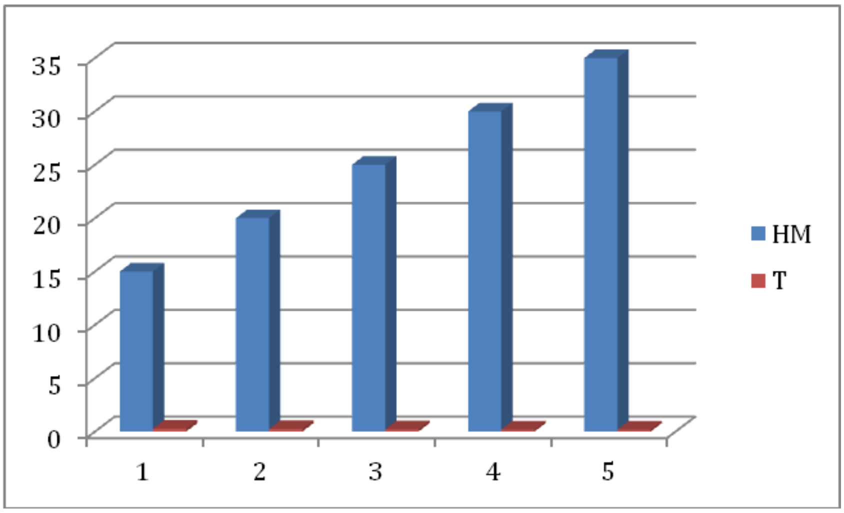
S.NO	H_M	$\tilde{O}_C$ = (35,40,45,50,55,60,65) $\tilde{D}_C$ = (54,64,74,84,94,104,114),		$\tilde{O}_C$ = (35,40,45,50,55,60,65) $\tilde{D}_C$ = (54,64,74,84,94,104,114),	
		GEOMETRIC PROGRAMMING		LAGRANGE METHOD	
		T	**TC(T)**	**T**	**TC(T)**
1	15	0.2817	354.97	0.2817	354.97
2	20	0.2440	409.88	0.2440	409.88
3	25	0.2182	458.26	0.2182	458.26
4	30	0.1992	501.99	0.1992	501.99
5	35	0.1844	542.22	0.1844	542.22

From the above table we observed that:

(i) The economic order time interval obtained by pascals Graded Mean Integration is equal to crisp economic order time interval
(ii) Total cost obtained by Graded Mean Integration is equal to crisp total cost

T value for various H_M values



TC value for various H_M values

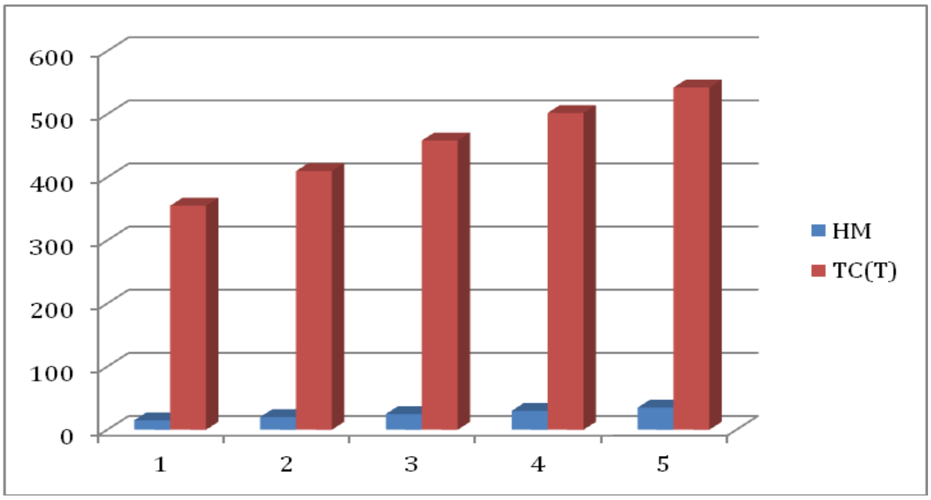


Figure 6.1 TC (T) and T value for various H_M values

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Conclusion

we have minimized the total cost and maximized optimum time interval Using non linear programming technique.such as geometric programming method and lagrange method, fuzzification was done by heptagonal fuzzy number and defuzzification was done by Pascal's Triangular Graded Mean .The comparison between two non linear programming methods,both the methods have the same result.we see that crisp and fuzzy values are also equal .

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