Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 4479 - 4484
Received 25 April 2021; Accepted 08 May 2021.

Applying Queuing Theory to Enhance the Service Provided by A Restaurant

L.K. Bhavani[1], G.Jayalalitha[2]

[1]Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies,Chennai, India. E-mail: bhavanitrix96@gmail.com

[2]Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai,India. E-mail: g.jayalalithamaths.sbs@velsuniv.ac.in

ABSTRACT

Queuing theory is the mathematical study of waiting lines, or queues. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. The common problem that arises in almost every famous restaurant is that they lose their customers due to a long wait on the line. This shows a need for a numerical model for the restaurant management to understand the situation better. This paper aims to show that queuing theory satisfies the model when tested with a real-case scenario. For instance the data from a restaurant "Dalchini" in Chennai is in order to derive the arrival rate, service rate, utilization rate, waiting time in queue and the probability of potential customers to balk. The collected data is analyzed by using Little's Theorem and M/M/1 queuing model. The arrival rate at "Dalchini restaurant", during its busiest period of the day is 3.25 customers per minute (cpm) while the service rate is 3.27 cpm during our study period. The average number of customers in the restaurant is 210 and the utilization period is 0.993.

KEYWORDS

Queuing Theory, Little's Theorem,Kendall's Notation,Waiting Lines,PoissonProcess.

Introduction

There are several determining factors for a restaurant to be considered a good or a bad one. Taste, cleanliness, the restaurant layout and settings are some of the most important factors. These factors, when managed carefully, will be able to attract plenty of customers. However, there is also another factor that needs to be considered especially when the restaurant has already succeeded in attracting customers. This factor is the customers queuing time. Once we are being served, our transaction with the service organization may be efficient, courteous and complete but the bitter taste of how long it took to get attention pollutes the overall judgments that we make about the quality of service. In a waiting line system, managers must decide what level of service to offer. A low level of service may be inexpensive, at least in the short run, but may incur high costs of customer dissatisfaction,such as lost future business and actual processing costs of complaints. Queuing theory is the study of queue or waiting lines.[1] Some of the analysis that can be derived using queuing theory include the expected waiting time in the queue, the average time in the system, the expected queue length, the expected number of customers served at one time, the probability of balking customers, as well as the probability of the system to be in certain states, such as empty or full. Waiting lines are a common sight in restaurants especially during lunch and dinner time.Hence, queuing theory is suitable to be applied in a restaurant setting since it has an associated queue or waiting line where customers who cannot be served immediately have to queue (wait) for service. Researchers have previously used queuing theory to model the restaurant operation [2], reduce cycle time in a busy fast food restaurant [3], as well as to increase throughput and efficiency [5]. This paper uses queuing theory to study the waiting lines in lines in "Dalchini", Restaurant at Chennai, and The restaurant provides 40 tables out of which some have 4 chairs and some have 6 chairs. There are 25 waiters working at any one time. On a daily basis it serves over 800 customers during weekdays and over 1300 customers during weekend. This paper seeks to illustrate the usefulness of applying queuing theory in a real case situation.

Queuing Theory

In 1908, Copenhagen Telephone Company requested Agner K. Erlang to work on the holding times in a telephone switch. He identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and exponentially distributed. This was the beginning of the study of queuing theory. In this section, we will discuss two common concepts in queuing theory.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 4479 - 4484
Received 25 April 2021; Accepted 08 May 2021.

Little's Theorem

Little's theorem [6] describes the relationship between through out rate (i.e. arrival and service rate), cycle time and work in process (i.e. number of customers/jobs in the system). This relationship has been shown to be valid for a wide class of queuing models. The theorem states that the expected number of customers (N) for a system in steady state can be determined using the following equation:

$$L = \lambda T \qquad (1)$$

Here, λ *is the average customer arrival rate and* T *is the average service time for a customer.*

Consider the example of a restaurant where the customer's arrival rate (λ) doubles but the customers still spend the same amount of time in the restaurant (T). These facts will double the number of customers in the restaurant (L).By the same logic, if the customer arrival rate (λ) remains the same but the customers service time doubles this will also double the total number of customers in the restaurant. This indicates that in order to control the three variables, managerial decisions are only required for any two of the three variables. Three fundamental relationships can be derived from Little's theorem [5]:

L increases if λ or T increases.
λ increases if L increases or T decreases.
T increases if L increases or λ decreases.

Rust [8] said that the Little's theorem can be useful in quantifying the maximum achievable operational improvements and also to estimate the performance change when the system is modified.

Queuing Models and Kendall's Notation

The principle actors in a queuing situation are the customer and the server. On arrival at a service facility, they can start service immediately or wait in a queue if the facility is busy. From the standpoint of analyzing queues, the arrival of customers is represented by the inter arrival time between successive customers, and the service is described by the service time per customer. The queuing behavior of customers plays a role in waiting line analysis. "Human" customers may jockey from one queue to another in the hope of reducing waiting time. They may also balk from joining a queue all together because of anticipated long delay, or they may renege from a queue because they have been waiting too long[8]. In most cases, queuing models can be characterized by the following factors:

1).Arrival Time Distribution

Inter-arrival times most commonly fall into one of the following distribution patterns: a Poisson distribution,a Deterministic distribution, or a General distribution. However, inter-arrival times are most often assumed to be independent and memory less, which is the attributes of a Poisson distribution[9].

2).Service Time Distribution

The service time distribution can be constant, exponential,hyper-exponential,hypo-exponential or general. The service time is independent of the inter-arrival time.

3).Number of Servers

The queuing calculationschange depends on whether there is a single server or multiple servers for the queue. A single server queue has one server for the queue. This is the situation normally found in a Book store where there is a line for each cashier. A multiple server queue corresponds to the situation in a bank in which a single line waits for the first of several tellers to become available.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 4479 - 4484
Received 25 April 2021; Accepted 08 May 2021.

4).Queue Lengths

The queue in a system can be modeled as having infinite or finite queue length.This includes the customers waiting in the queue[10].

5).Queuing Discipline

There are several possibilities in terms of the sequence of customers to be served such as FIFO (First In First Out, i.e. in order of arrival), random order, LIFO (Last In First Out, i.e. the last one to come will be the first to be served), SIRO (Service in Random Order).

6).System Capacity

The maximum number of customers in a system can be from 1 - infinity.Kendall, in 1953, proposed a notation system to represent the six characteristics discussed above. The notation of a queue is written as: A/B/P/Q/R/Z. A *describes the distribution type of the interarrival times,* B *describes the distribution type of the service times,* P *describes the number of servers in the system,*
Q *(optional) describes the maximum length ofthe queue,*
R *(optional) describes the size of the system population and*
Z *(optional) describes the queuing discipline*

Dalchini Queuing Model

The daily number of visitors was obtained from the restaurant itself. The restaurant has been recording the data as part of its end of day routine.In general from the data that is obtained, we concluded that the queuing model that best illustrate the operation of "Dalchini" is M/M/1.This means that the arrival and service time are exponentially distributed (Poisson process). The restaurant system consists of only one server. In our observation the restaurant has several waitresses but in the actual waiting queue, they only have one chef to serve all of the customers. Figure 1 illustrates the M/M/1 queuing model.

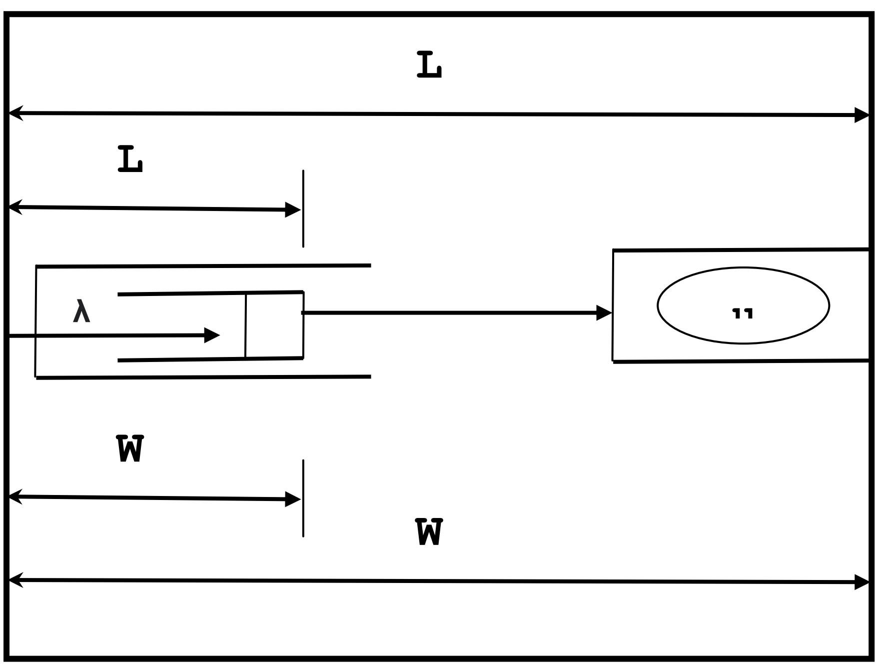

Fig. 1.M/M/1 Queuing Model

For the analysis of the "Dalchini" M/M/1 Queuing Model, the following variables will be investigated [6]:
λ: *The mean customers arrival rate*
μ: *The mean service rate*

ρ: λ/μ: *utilization factor*

Probability of zero customers in the restaurant(Po) is given by

$$Po = 1 - \rho \quad (2)$$

Pn: *The probability of having n customers in the restaurant.*

$$Pn = Po\rho n = (1 - \rho)\rho n \quad (3)$$

L: *average number of customers dining in the restaurant.*

$$L = \rho/1\text{-}\rho = \lambda/\mu\text{-}\lambda \quad (4)$$

Lq: *average number of customers in the queue.* $Lq = \rho 2/1-\rho$ (5)

W: *average time spent in **Dalchini** including the waiting time.*

$$W = 1/\mu\text{-}\lambda \quad (6)$$

Wq: *average waiting time in the queue*.

$$Wq = Lq/\lambda \quad (7)$$

Data Analysation

The one week analysis is shown by graph below analyzed by us at dinner slot.

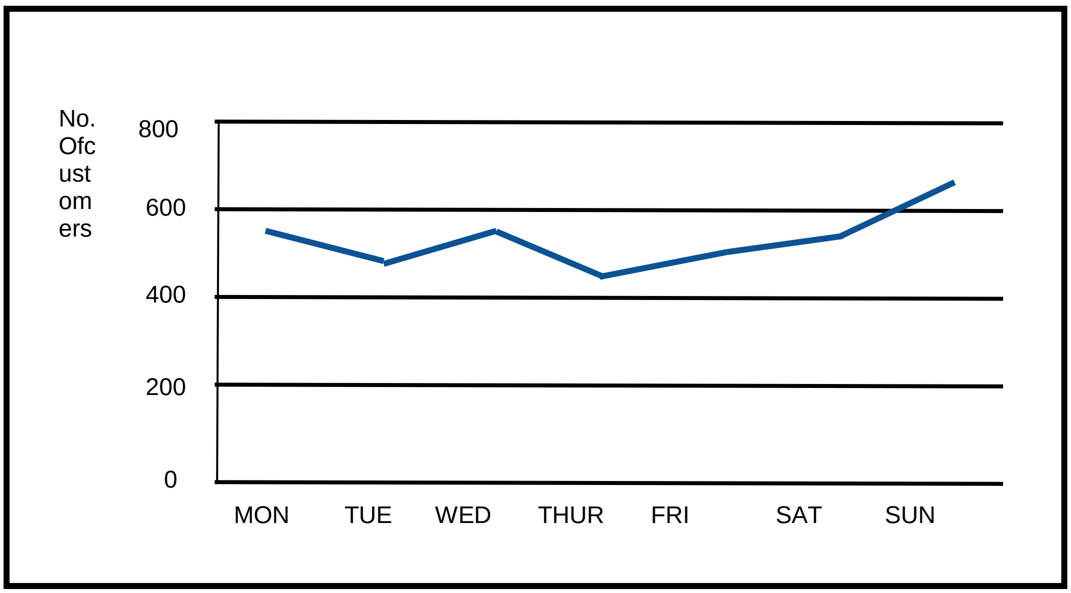

Fig.2. One Week Analysis of Number of Customers arriving in restaurant

As can be seen in Figure 2, the number of customers on Saturdays and Sundays are double. The busiest period for the restaurant is on weekend during dinner time. Hence focus our analysis in this time window. Authors analyzed the restaurant between 19 to 22 hours.

Calculation

Our teams conducted the research at dinner time. There are on average 585 people are coming to the restaurant in 3 hours time window of dinner time. From this we can derive the arrival rate as:

$$\lambda = 585/180$$
$$= 3.25 \text{ customer per minute (cpm)}$$

We also found out from observation and discussion with the manager that each customer spends 65 minutes on average in the restaurant (W), *the queue length is around 35 people* (Lq) on average and the waiting time is around 12 minutes. It can be shown using (7) that the observed actual waiting time does not differ by much when compared to the theoretical waiting time as shown below

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 4479 - 4484
Received 25 April 2021; Accepted 08 May 2021.

Wq = Lq/λ
= 35 customers/3.25cpm
= 10.76 minutes

Next, calculate the average number of people in the restaurant using the above calculated values,
L = 3.25 cpm × 65 minutes
= 211 customers

After calculating the average number of customers in the restaurant "**Dalchini**",derive the service rate as:
μ =λ(1 + L)/L
= 3.25(1 + 211)/211
= 3.27 cpm(approx)

Now, calculate Traffic Intensity or utilization factor
ρ = λ/μ
= 3.25/3.27 = 0.993

With the high utilization rate of 0.993 during dinner time the probability of zero customers in the restaurant or probability that system is idle can be calculated by (2)
Po = 1 − ρ = 0.007

The generic formula that can be used to calculate the probability of having 'n' customer in the restaurant is as follows:
Pn= (1 − ρ)ρn = (1 − 0.993)0.993^n
= (0.007)$(0.993)^n$

Assume that potential customers will start to balk when they see more than 30 people are already queuing for the restaurant and the maximum queue length that a potential customer can tolerate is 40 people. As the capacity of the restaurant when fully occupied is 200 people, can calculate the probability of 30 people in the queue as the probability when there are 240 people in the system (i.e. 210 in the restaurant and 30 or more queuing) as follows:
Probability of customers going away = P (more than 30 people in the queue) = P (more than 240)people in the restaurant.

$$P211-240 = \sum_{n=211}^{240} Pn = \sum_{n=211}^{240} (.007)(0.993)^n = 4.31\%$$

Analysis

The utilization is directly proportional with the mean number of customers. It means that the mean number of customers will increase as the utilization increases. The utilization rate at the restaurant is very high at 0.993. This, however,is only the utilization rate during lunch and dinner time on Saturdays and Sundays. On weekdays, the utilization rate is almost half of it. This is because the number of visitors on weekdays is only half of the number of visitors on weekends.
In addition, the number of waiters or waitresses remains the same regardless whether it is peak hours or off-peak hours. In case the customers waiting time is lower or in other words we waited for less than 15 minutes, the number of customers that are able to be served per minute will increase. When the service rate is higher the utilization will be lower, which makes the probability of the customers going away decreases.

Benefits

This research can help "**Dalchini**" to increase their QOS (Quality of Service), by anticipating if there are many customers in the queue. The result of this paper work may become the reference to analyze the current system and improve the next system. Because the restaurant can now estimate how many customers will wait in the queue and the number of customers that will go away each day.By anticipating the huge number of customers coming and going in a day, the restaurant can set a target profit that should be achieved daily and the formulas that were used during the completion of the research is applicable for future research and also could be used to develop more complex theories.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 4479 - 4484
Received 25 April 2021; Accepted 08 May 2021.

Conclusion

This study has discussed the application of queuing theory of "**Dalchini**" Restaurant. Here authors focused on two particularly common decision variables as a medium for introducing and illustrating all the concepts. From the result authors obtained that the rate at which customers arrive in the queuing system is 3.25 customers per minute and service rate is 3.27 cpm and utilization rate is 0.993. This theory is also applicable for the restaurant if they want to calculate all the data daily. It can be concluded that the arrival rate will be lesser and the service rate will be greater if it is on weekdays since the average number of customers is less as compared to those on weekends. The constraints that were faced for the completion of this research were the inaccuracy of the result since some of the data that was just based on assumption or approximation. Authors hope that this research can contribute to the betterment of **Dalchini** restaurant in terms of its way of dealing with customers.

Future Outcomes

This study will develop a simulation model for the restaurant which will be able to confirm the results of the analytical model. In addition, a simulation model allows adding more complexity so that the model can mirror the actual operation of the restaurant more closely. This study gives a generalized guarantee to stabilize the system from the problems arisen like customers' balking, reneging, jockeying and collusion or delay in services by the present way of working in a restaurant. In today's world of accelerating advancement in computer technology, it will be fruitful for restaurant managers to install a computer for the proper control of service facilities and to keep the previous record so as to make the forecasting better over good in order to excel in the field.

References

[1] A. K. Kharwat, *Computer Simulation IEEE Press*,pp. 811-815,(1991).

[2] M.Laguna and J. Marklund, *Pearson Prentice Hall* ISBN, (2005).

[3] J. D. C. Little, "A Proof for the Queuing Formula: L= λW," Operations Research vol. 9(3), pp. 383-387,1961.

[4] K. Rust, *Using Little's Law to Estimate Cycle Time and Cost,* IEEE Press 2008.

[5] Sonali Yadav et.al, *Optimization of Quality of Service of Restaurant*, Academic Press 2012.

[6] T. Altiok and B. Melamed, *Simulation Modeling and Analysis with ARENA*,Academic Press,ISBN0-12-370523 1,2007.

[7] D.M. Brann and B.C. Kulick, *Simulation of restaurant operations using the Restaurant Modeling Studio*,IEEE Press 2002.

[8] V. Sundaresan, and K.S. Ganapathy Subramanian and K. Ganesan, A.R. Publications, pp. 13.1-13.8

[9] Hamdy A. Taha, *University of Arkansas, Fayetteville,* Pearson Prentice Hall pp.556-563,2003.

[10] S.D. Sharma, *Operations Research Kedarnath Ramnath & Co publication,* pp. 42-56, 2002.

[11] Hamdy A. Taha,*University of Arkansas, Fayetteville,Pearson Prentice Hall* pp.556-563,2003.

[12] S.D.Sharma,*OperationsResearchKedarnath Ramnath & Co publication,*pp. 42-56, 2002.