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Influence of Noise Effect on a Peculiar Ecostystem

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ABSTRACT

The paper intends to investigate a peculiar ecosystem. It contains Ammensal and Enemy species with limited resources. Mortality and immigration are both applied on Ammesnal Species. Global stability is identified by choosing suitable Liapunov's function. Stochastic Analysis has been employed. Series solutions are provided with the help of Homotopy perturbation method.

Key words:
Ammensal, Enemy Species, Stability, Local Stability, HPM, global stability, *Gaussian white noise.*

1. Introduction

Mathematical Modelling may be viewed as a interdisciplinary concept that deals with the relationship of mathematics and other disciplines. An analytical practice deals with many facets of the everyday environment. The phases are described as i).Defining the issue in the real world particularly in the biological or medical or social sense.(ii) prediction model formulation. (iii) Solving math problems that can emerge when evaluating the model.(iv) Development of analysis techniques and similar computer programmes for the computations involved.(v) To clarify and to see the outcomes in the context of the original issue and to convey this knowledge to all needy people. With an effective procedure of knowledge processing, mathematical models have been valuable methods in biological investigations. If these models are built and used appropriately, they can give insight into the interactions between the physical factors and the mechanism that influences the structure being examined. A major component in the design of experiments and in the analysis of results may be the subsequent interplay between experimental inquiry and the theoretical model.

There are two kinds of mathematical structures in general: Deterministic and Stochastic. The formulation of the processes in deterministic models relies on various axioms / hypotheses to be considered due to the relevant system biology and these may be provided in the form of autonomous or non-autonomous, ordinary / partial (linear or nonlinear) differential or integro-differential equations. In general, stochastic models are probabilistic models. K.V.L.N.Acharyulu and N.Ch. Pattabhi Ramacharyulu[5-18] examined local and global ecosystem equilibrium with multiple dimensions. In the earlier work, local stability was conducted for various Ammensal-Enemy eco-systems with diverse tools. The present investigation focuses primarily on establishing the global stability, Local stability and series solution and the authors investigated various ecological models for their stability. Many Research scholars [1-4] and Mathematicians [19-31] extended their significant contributions to this modelling field.

1.1 Notations:

This is an evolutionary environment where Ammensal and Enemy species live together. It is believed that all interacting ecological species are continuously harvested (migrated or immigrated) by depending upon available natural resources. Here the Ammensal species is effected by mortality and strengthened by harvesting

(i).X represents the density of Ammensal species at natural growth rate a_1.

(ii).Y stands for the density of the Enemy species,
(iii).$h_1(=a_{11}H_1)$ is the harvesting of Ammensal species,
(iv). $K_i(=\frac{a_i}{a_{ii}})$ be the carrying capacity of Ammensal Species .
(v). $\alpha(=\frac{a_{12}}{a_{11}})$ be the Ammensalism's coefficient.
Assume that the parameters described above are positive.

2. Constriction of Mathematical Model

The rate of growth equation for the Ammensal species with constant rates of mortality and harvesting:

$$\frac{dX}{dt}=a_{11}(-K_1X-X^2-\alpha XY+H_1) \qquad (2.1)$$

The rate of growth equation for the Enemy species :

$$\frac{dY}{dt}=a_{22}Y(K_2-Y) \qquad (2.2)$$

In this model , the interior point is obtained as

$$E_4(x^*,y^*)\text{ where } x^*=\frac{\sqrt{4H_1+(K_1+\alpha K_2)^2}-(\alpha K_2+K_1)}{2},\ y^*=K_2$$

3. Global Stability of The System by Lyapunov Property

Liapunov has developed a valuable tool to efficiently assess global stability.

Theorem (4.1): The constituted special ecosystem (2.1)-(2.2) is globally asymptotically stable at the positive equilibrium(x^*,y^*).

Proof: Now construct suitable Liapunov function to address the global stability at interior equilibrium $E_4(x^*,y^*)$

$$V(t)=\left((x-x^*)-x^*(\ln x-\ln x^*)\right)+l_1\left((y-y^*)-y^*(\ln y-\ln y^*)\right),l_1>0 \qquad (3.1)$$

$$\frac{dV}{dt}=\left(\frac{x-x^*}{x}\right)\frac{dx}{dt}+l_1\left(\frac{y-y^*}{y}\right)\frac{dy}{dt}$$

$$=\left(\frac{x-x^*}{x}\right)\left[a_{11}x(-k_1-x-\alpha y+\frac{H_1}{x})\right]+l_1\left(\frac{y-y^*}{y}\right)\left[a_{22}y(k_2-y)\right]$$

$$=-a_{11}(x-x^*)^2-(y-y^*)^2-H_1(\frac{(x-x^*)^2}{xx^*})-\alpha a_{11}(y-y^*)(x-x^*)$$

$$\frac{dV}{dt}\le -a_{11}(x-x^*)^2-(y-y^*)^2-H_1(\frac{(x-x^*)^2}{xx^*})-\frac{\alpha a_{11}}{2}\left((y-y^*)^2+(x-x^*)^2\right)$$

$$\le -(x-x^*)^2\left(a_{11}+\frac{H_1}{xx^*}+\frac{\alpha a_{11}}{2}\right)-(y-y^*)^2\left(1+\frac{\alpha a_{11}}{2}\right)$$

$$\frac{dV}{dt}\le -\left((x-x^*)^2\left(a_{11}+\frac{H_1}{xx^*}+\frac{\alpha a_{11}}{2}\right)+(y-y^*)^2\left(1+\frac{\alpha a_{11}}{2}\right)\right)<0$$

Provided $\left(a_{11}+\frac{H_1}{xx^*}+\frac{\alpha a_{11}}{2}\right)>0 \quad and \quad \left(1+\frac{\alpha a_{11}}{2}\right)>0$

The condition $V'(t)<0$ holds. hence the non-diffusive system is asymptotically stable .

4. Stochastic Analysis of The Model

Stochastic analysis is based on the statistical calculus. This calculation was developed in order to resolve issues resulting from the theory of probability in which systems are driven along paths that cannot usually be separated. Stochastic analysis is a fundamental method of many modern probability theories with statistical inferences and is used in many fields of application from biology to physics.

The equations of special ecosystem with noise effect on (2.1)-(2.2) are

$$\frac{dX}{dt} = a_{11}(-K_1X - X^2 - \alpha XY + H_1) + \alpha_1\xi_1(\mathrm{t}) \quad (4.1)$$

$$\frac{dY}{dt} = a_{22}Y(K_2 - Y) + \alpha_2\xi_2(\mathrm{t}) \quad (4.2)$$

here α_1, α_2 stands for real constants , the Gaussian white noise effect : $\xi_i(t) = [\xi_1(t), \xi_2(t)]$ is in a two dimensional system with the conditions $E(\xi_i(t)) = 0; i = 1,2;$

$v = \delta_{ij}\delta(t - t'); i = j = 1,2$ where δ_{ij} is the Kronecker delta function; δ is the Dirac –delta function.

By the concept of Nisbet and Gurney [21], Gaussian white noise effect at the interior equilibrium point $E_4(x^*, y^*)$ is discussed by taking perturbations

$X(t) = u_1(t) + S^*$ and $Y(t) = u_2(t) + P^*$

Hence, the model (4.1)-(4.2) reduces to the following linear system and
The linear part of the system (4.1)-(4.2) is

$$\frac{du_1}{dt} = -a_{11}S^*(u_1 + u_2) + \alpha_1\xi_1(\mathrm{t}) \quad (4.3)$$

$$\frac{du_2}{dt} = a_{22}P^*u_2 + \alpha_2\xi_2(\mathrm{t}) \quad (4.4)$$

Taking the Fourier transform of (4.3) and (4.4) we get,

$$\alpha_1\tilde{\xi}_1(\omega) = \left(i\omega + a_{11}S^*\right)\tilde{u}_1(\omega) + a_{22}S^*\tilde{u}_2(\omega) \quad (4.5)$$

$$\alpha_2\tilde{\xi}_2(\omega) = \left(i\omega - a_{22}P^*\right)\tilde{u}_2(\omega) \quad (4.6)$$

Now, represent (4.5) and (4.6) in a standard matrix form as $M(\omega)\tilde{u}(\omega) = \tilde{\xi}(\omega)$ (4.7)

where $M(\omega) = \begin{pmatrix} A(\omega) & B(\omega) \\ C(\omega) & D(\omega) \end{pmatrix}; \tilde{u}(\omega) = \begin{bmatrix} \tilde{u}_1(\omega) \\ \tilde{u}_2(\omega) \end{bmatrix}; \tilde{\xi}(\omega) = \begin{bmatrix} \alpha_1\tilde{\xi}_1(\omega) \\ \alpha_2\tilde{\xi}_2(\omega) \end{bmatrix};$

$$A(\omega) = i\omega + a_{11}S^*; B(\omega) = a_{22}S^*; C(\omega) = 0; D(\omega) = i\omega - a_{22}P^* \quad (4.8)$$

Hence the solution of (4.7) is given by $\tilde{u}(\omega) = K(\omega)\tilde{\xi}(\omega)$,

where $K(\omega) = \left[M(\omega)\right]^{-1}$ (4.9)

The solutions of (4.9) are given by

$$\tilde{u}_i(\omega) = \sum_{j=1}^{2} K_{ij}(\omega)\tilde{\xi}_j(\omega); \; i = 1,2 \quad (4.10)$$

The spectrum of u_i, $i = 1,2$ are given by $S_{u_i}(\omega) = \sum_{j=1}^{2}\alpha_j\left|K_{ij}(\omega)\right|^2; i = 1,2$

Intensities of fluctuations of the component u_i, $i = 1,2$ are provided by $\sigma_{u_i}{}^2 = \frac{1}{2\pi}\sum_{j=1}^{2}\int_{-\infty}^{\infty}\alpha_j\left|K_{ij}(\omega)\right|^2 d\omega; \; i = 1,2$

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From (4.10), we obtain $\sigma_{u_1}{}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\alpha_1\left|\frac{D(\omega)}{|M(\omega)|}\right|^2 d\omega + \int_{-\infty}^{\infty}\alpha_2\left|\frac{B(\omega)}{|M(\omega)|}\right|^2 d\omega\right\}$

$$\sigma_{u_2}{}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty}\alpha_1\left|\frac{A(\omega)}{|M(\omega)|}\right|^2 d\omega + \int_{-\infty}^{\infty}\alpha_2\left|\frac{C(\omega)}{|M(\omega)|}\right|^2 d\omega\right\}$$

where $|M(\omega)| = R(\omega) + iI(\omega)$; $R(\omega) = -(\omega^2 + a_{11}a_{22}S^*P^*)$; $I(\omega) = \omega(a_{11}S^* - a_{22}P^*)$

If we take into consideration the noise effect on one of the species $\alpha_1 = 0$ *or* $\alpha_2 = 0$

then we have

$$If\ \alpha_1 = 0 \text{ and } \sigma_{u_1}{}^2 = \frac{\alpha_2(a_{22}S^*)^2}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}d\omega \ \&\ \sigma_{u_2}{}^2 = 0$$

$$If\ \alpha_2 = 0 \ \&\ \sigma_{u_1}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\omega^2 + (a_{22}P^*)^2\right]d\omega \ \&\ \sigma_{u_2}{}^2 = 0$$

Population variances suggest population stability with smaller mean square fluctuations, whereas greater population variance values indicate population instability.

5. Series Solutions by Homotopy Perturbation Method (HPM)

HPM is a effective and valuable technique for discovering series solutions of non linear equations without a linearization procedure. He first implemented the process efficiently. HPM incorporates perturbation and homotopy processes. This approach will take advantage of traditional perturbation method thus avoiding constraints. In general, several mathematicians used this approach successfully to solve all kinds of linear and nonlinear equations in Science, Engineering and Technology.

By the definition of homotopy, the following structure can be designed as

$$\beta_1^1 - X_0{}^1 + \psi(X_0{}^1 + a_1\beta_1 + a_{11}\beta_1^2 + a_{12}\beta_1\beta_2 - a_{11}H_1) = 0$$

$$\beta_2^1 - Y_0^1 + \psi(Y_0^1 - a_2\beta_2 + a_{22}\beta_2{}^2) = 0$$

$$Assume\ \beta_1 = \beta_{1,0}(t)\, and\ \beta_2 = \beta_{2,0}(t) \tag{5.1}$$

The first approximations are taken into account as

$$\beta_{1,0}(t) = \beta_1(0) = X_0(t) = \lambda_1 \text{ and } \beta_{2,0}(t) = \beta_2(0) = Y_0(t) = \lambda_2 \tag{5.2}$$

$$\beta_1(t) = \beta_{1,0}(t) + \psi\,\beta_{1,1}(t) + \psi^2\beta_{1,2}(t) + \psi^3\beta_{1,3}(t) + \psi^4\beta_{1,4}(t) + \psi^5\beta_{1,5}(t) + \ldots\ldots \tag{5.3}$$

$$\beta_2(t) = \beta_{2,0}(t) + \psi\,\beta_{2,1}(t) + \psi^2\beta_{2,2}(t) + \psi^3\beta_{2,3}(t) + \psi^4\beta_{2,4}(t) + \psi^5\beta_{2,5}(t) + \ldots\ldots \tag{5.4}$$

Here $\beta_{i,j}$ $(i = 1,2\ ;\ j = 1,2,3\ldots)$,to be decided by the substitution of(5.1),(5.2),(5.3)&(5.4)

Now comparing the coefficient of various powers of ψ in the above approximations

After simplification, various coefficients are obtained as below

The coefficient of ψ^1 :

$$\beta_{1,1}{}^1(t) + a_1\,\beta_{1,0}(t) + a_{11}\,\beta^2{}_{1,0}(t) + a_{12}\beta_{1,0}(t)\,\beta_{2,0}(t) - a_{11}H_1 = 0$$

$$\beta_{2,1}{}^1(t) - a_2\,\beta_{2,0}(t) + a_{22}\,\beta^2{}_{2,0}(t) = 0$$

The coefficient of ψ^2 :

$$\beta_{1,2}{}^1(t) + a_1\,\beta_{1,1}(t) + 2a_{11}\,\beta_{1,0}(t)\,\beta_{1,1}(t) + a_{12}\,\beta_{1,0}(t)\,\beta_{2,1}(t) + a_{12}\,\beta_{1,1}(t)\,\beta_{2,0}(t) = 0\, and$$

$$\beta_{2,2}{}^1(t) - a_2\,\beta_{2,1}(t) + 2a_{22}\,\beta_{2,0}(t)\,\beta_{2,1}(t) = 0$$

The coefficient of ψ^3 :

$$\beta_{1,3}{}^1(t) + a_1\beta_{1,2}(t) + 2a_{11}\beta_{1,0}(t)\beta_{1,2}(t) + a_{11}\beta^2{}_{1,1}(t)$$

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$+a_{12}\beta_{1,0}(t)\beta_{2,2}(t)+a_{12}\beta_{1,1}(t)\beta_{2,1}(t)+a_{12}\beta_{1,2}(t)\beta_{2,0}(t)=0$ *and*

$\beta_{2,3}{}^{1}(t)-a_2\,\beta_{2,2}(t)+2a_{22}\,\beta_{2,0}(t)\,v_{2,2}(t)+a_{22}\,\beta_{2,1}{}^{2}(t)=0$

The coefficient of ψ^4 :

$\beta_{1,4}{}^{1}(t)+a_1\,\beta_{1,3}(t)+2a_{11}\,\beta_{1,0}(t)\,\beta_{1,3}(t)+2a_{11}\,\beta_{1,1}(t)\,\beta_{1,2}(t)$

$+a_{12}v_{1,0}(t)v_{2,3}(t)+a_{12}v_{1,1}(t)v_{2,2}(t)+a_{12}v_{1,2}(t)v_{2,1}(t)++a_{12}v_{1,3}(t)v_{2,0}(t)=0$ *and*

$\beta_{2,4}{}^{1}(t)-a_2\,\beta_{2,3}(t)+2a_{22}\,\beta_{2,0}(t)\beta_{2,3}(t)+2a_{22}\,\beta_{2,1}(t)\,\beta_{2,2}(t)=0$

Now $\beta_{1,1}(t)=-a_1\int_0^t \beta_{1,0}(t)\,dt\,-a_{11}\int_0^t \beta^2{}_{1,0}(t)\,dt-a_{12}\int_0^t \beta_{1,0}(t)\beta_{2,0}(t)\,dt+a_{11}\int_0^t H_1\,dt$

$\therefore \beta_{1,1}\left(t\right)=(-a_1\lambda_1-a_{11}\lambda_1{}^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)t$

$\beta_{2,1}(t)=\,a_2\int_0^t \beta_{2,0}(t)\,dt\,-a_{22}\int_0^t \beta^2{}_{2,0}(t)\,dt$

$\therefore\;\beta_{2,1}(t)=\,(a_2\lambda_2-a_{22}\lambda_2{}^2)t$

$\beta_{1,2}(t)=\left(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\right)\int_0^t \beta_{1,1}(t)\,dt\,-a_{12}\lambda_1\int_0^t \beta_{2,1}(t)\,dt$

$\therefore \beta_{1,2}(t)=\left[\left(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\right)(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)\right]\frac{t^2}{2}$

$\beta_{2,2}(t)=\left(a_2-2a_{22}\lambda_2\;\right)\int_0^t \beta_{2,1}(t)\,dt$

$\therefore \beta_{2,2}\left(t\right)\;=[\left(a_2-2a_{22}\lambda_2\right)(a_2\lambda_2-a_{22}\lambda_2^2)]\frac{t^2}{2}\;\;\beta_{1,3}(t)=-a_1\int_0^t \beta_{1,2}(t)\,dt-2a_{11}\int_0^t \beta_{1,0}(t)\beta_{1,2}(t)\,dt-a_{11}\int_0^t \beta^2{}_{1,1}(t)\,dt$

$-a_{12}\int_0^t \beta_{1,0}(t)\beta_{2,2}(t)\,dt\,-a_{12}\int_0^t \beta_{1,1}(t)\beta_{2,1}(t)\,dt-a_{12}\int_0^t \beta_{1,2}(t)\beta_{2,0}(t)\,dt$

$\Rightarrow \beta_{1,3}(t)=\left(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\;\right)\int_0^t \beta_{1,2}(t)\,dt$

$-\left(a_{11}\int_0^t \beta_{1,1}(t)\,dt\,+a_{12}\int_0^t \beta_{2,1}(t)\,dt\right)\int_0^t \beta_{1,1}(t)\,dt\,-\beta_{12}\psi_1\int_0^t \beta_{2,2}(t)\,dt$

$\therefore \beta_{1,3}(t)\;=\Big\{\Big[\left(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\right)\Big[\left(-a_1-2a_{11}\lambda_1-a_{12}\,\lambda_2\right)(-a_1\lambda_1-a_{11}\,\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)$

$-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)\Big]-\Big[a_{11}(-a_1\lambda_1-a_{11}\lambda_1{}^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)+a_{12}(a_2\lambda_2-a_{22}\lambda_2{}^2)\Big]$

$(-a_1\lambda_1-a_{11}\,\lambda_1{}^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-a_{12}\lambda_1\left(a_2-2a_{22}\beta_2\right)(a_2v_2-a_{22}\,\lambda_2^2)\Big\}\frac{t^3}{6}$

Similarly $\beta_{2,3}\left(t\right),\beta_{1,4}(t)$ are calculated

The approximations of 4-terms are adequate, hence, we have

$$X(t)=\lim_{\psi\to1}\beta_1(t)=\sum_{x=0}^{4}\beta_{1,x}(t)=\beta_{1,0}(t)+\psi\,\beta_{1,1}(t)+\psi^2\beta_{1,2}(t)+\psi^3\beta_{1,3}(t)+\psi^4\beta_{1,4}(t)$$

$$Y(t)=\lim_{\psi\to1}\beta_2(t)=\sum_{x=0}^{4}\beta_{2,x}(t)=\beta_{2,0}(t)+\psi\,\beta_{2,1}(t)+\psi^2\beta_{2,2}(t)+\psi^3\beta_{2,3}(t)+\psi^4\beta_{2,4}(t)$$

The series solutions are derived with the help of HPM as

$$X(t)=\lambda_1+(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)t$$
$$+\Big[\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)\Big]\frac{t^2}{2}$$
$$+\Big\{\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)\Big[\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)$$
$$-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)\Big]-a_{11}(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)$$
$$-a_{12}(a_2\lambda_2-a_{22}\lambda_2^2)\Big[(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)\Big]-a_{12}\lambda_1(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big\}\frac{t^3}{6}$$
$$+\Big\{\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)\Big[\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)\Big[\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)$$
$$(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)\Big]$$
$$-a_{11}(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-a_{12}(a_2\lambda_2-a_{22}\lambda_2^2)$$
$$\Big[(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)\Big]-a_{12}\lambda_1(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big]$$
$$\Big[-6a_{11}(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)-3a_{12}(a_2\lambda_2-a_{22}\lambda_2^2)\Big]$$
$$\Big[\big(-a_1-2a_{11}\lambda_1-a_{12}\lambda_2\big)\Big[(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)\Big]-a_{12}\lambda_1(a_2\lambda_2-a_{22}\lambda_2^2)$$
$$-a_{12}\lambda_1\Big[(a_2-2a_{22}\lambda_2)\Big[(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big]-a_{22}\big(a_2\lambda_2-a_{22}\lambda_2^2\big)^2\Big]$$
$$-3a_{12}(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)(-a_1\lambda_1-a_{11}\lambda_1^2-a_{12}\lambda_1\lambda_2+a_{11}H_1)\Big\}\frac{t^4}{24}+\ldots\ldots\ldots\ldots$$

$$Y(t)=\lambda_2+(a_2\lambda_2-a_{22}\lambda_2^2)t+(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\frac{t^2}{2}$$
$$+\Big\{\big(a_2-2a_{22}\lambda_2\big)\Big[(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big]-a_{22}(a_2\lambda_2-a_{22}\lambda_2^2)^2\Big\}\frac{t^3}{6}$$
$$+\Big\{\big(a_2-a_{22}\lambda_2\big)\Big[\big(a_2-2a_{22}\lambda_2\big)\Big[(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big]-a_{22}(a_2\lambda_2-a_{22}\lambda_2^2)^2\Big]$$
$$-3a_{22}(a_2\lambda_2-a_{22}\lambda_2^2)(a_2-2a_{22}\lambda_2)(a_2\lambda_2-a_{22}\lambda_2^2)\Big\}\frac{t^4}{24}+\ldots\ldots.$$

6. Conclusions

Based on the study of Migrated Ammensal Model, the following Conclusions have been observed:

(i).Global Stability is achieved by constructing proper Lyapunov function. The necessary theorems for global stability are established.

(ii).The stochastic Analysis is employed successfully for identifying the impact of smaller mean square fluctuations on the stability.

(iii).The series solutions with possible higher degrees are derived.

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